A new volume-preserving and continuous interface reconstruction method for multimaterial flow

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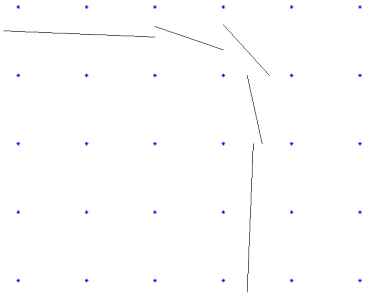
Outline of the talk

Main objectives of the work

- The objective of this work is to develop a volume preserving and continuous interface reconstruction (IR) method to avoid numerical artefacts in multimaterial computations.
- The new IR method must be able to avoid some drawbacks of classical methods like the Youngs method :



Desired properties of the IR method

- The following properties are mandatory for the new interface reconstruction method :
  - P1 : volumic fractions conservation
  - P2 : continuity of the interface
  - P3 : robustness
  - P4 : low cost
- Other properties may also be interesting :
  - C5 : 3D capability
  - C6 : compatibility with unstructured meshes
  - C7 : extension to more than 2 materials
  - C8 : parallelisation.

Properties of some existing IR methods

- The following table summarizes the properties of various existing IR methods :

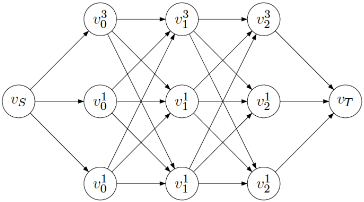
	P1 (vol.)	P2 (cont.)	P3 (robust.)	P4 (cost)
Youngs	yes	no	yes	yes
Youngs/Dilts	yes	yes	no	no
MOF	yes	no	yes	no
level set	yes	yes	no	yes

Main principle of Dynamic programming (DP)

- Dynamic programming is a fast optimization method applied to cost functions on discrete sets of the following type :

$$J(x_1, ..., x_N) = \sum_{i=0}^{N-1} f_{i,i+1}(x_i, x_{i+1}).$$

- It is based on algorithm that computes the shortest path in an associated graph, which exploits its structure, at cost $O(NL^2)$.



- Dynamic Programming is well known in imagery to solve segmentation problems.

Application of DP to interface reconstruction

- The interface reconstruction problem can be seen as a minimization problem of the total sum of volumic fractions errors :

$$J(y) = \sum_{(i,j)\in\{1,\ldots,N_x\}\times\{1,\ldots,N_y\}} |vol_{y,i,j} - vol_{i,j}|^p$$

where $t \mapsto y(t)$ is the associated interface curve.

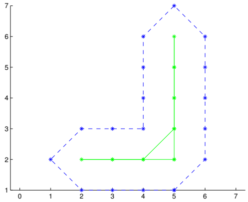
- The curve $t \mapsto y(t)$ that has to be found here must be continuous and satisfy $J(y) = 0$.

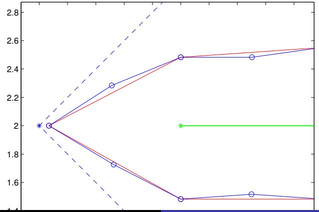
Main principle of the new IR method

- In our case, the closed curved $t \mapsto y(t)$ is supposed to be continuous and piecewise linear on each cell.
- The cost function J can then be minimized with a dynamic programming method, as it has the correct type.
- A regularization term such as $\lambda|y|$ can also be added to the cost function J to reduce wave effects.
- After DP minimization, a second step is added in order to make vanish the remaining error.
- It is done locally by adding an additional control point on each cell.

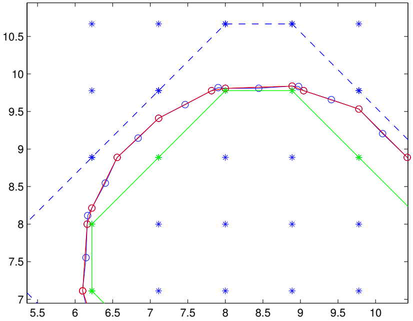
A very simple example

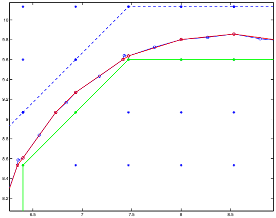
				.10	.10	
				.25	.25	
				.25	.25	
				.5	.4	
				.4	.5	
				.5	.5	
	.25	.5	.60	.75	.25	
	.25	.5	.40	.25	.10	



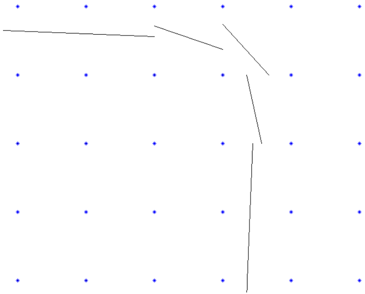


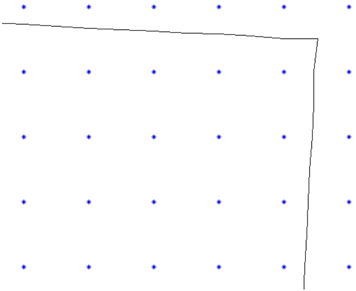
Example of a disc reconstruction





A square reconstruction : comparison with Youngs method

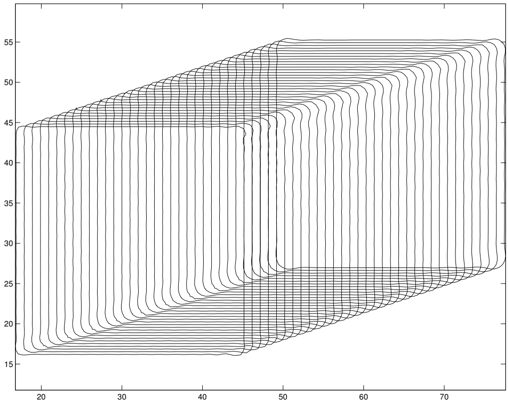




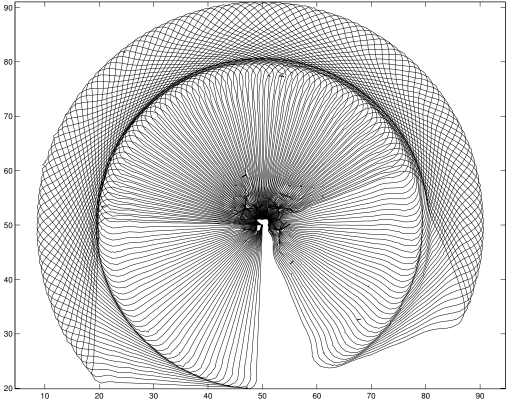
Coupling with advection schemes

- The new IR method is tested in the context of a Lagrange+ remap advection scheme with a given advection field.
- It has been compared with a Youngs reconstruction method for the following test cases :
  - Constant advection of a disc or a square
  - Square in a rotating field
  - Disc in a single vortex field
- Note that the new IR method has also successfully be coupled with diffusion schemes in the case of multimaterial diffusion (not presented here).

Constant advection of a square with the new IR method

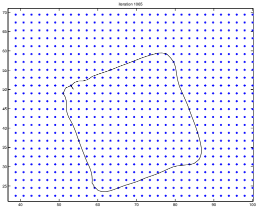


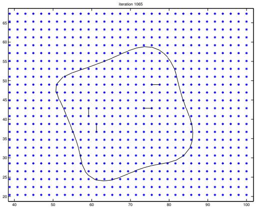
Rotation of a square with the new IR method



Comparison with a Youngs reconstruction

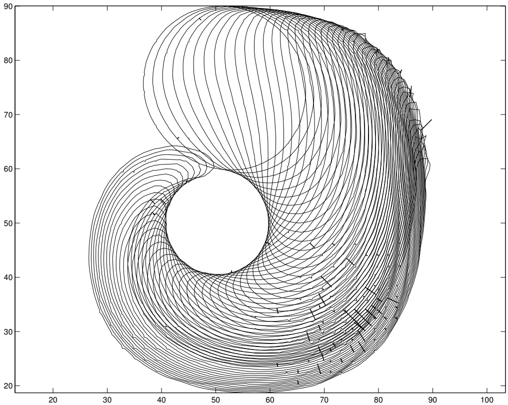
- The figures below compares the deformation of the square after a complete rotation (left : new IR method, right : Youngs) .





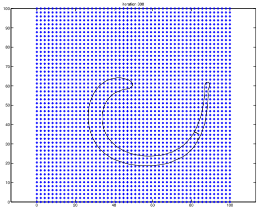
- It can be seen that sharp angles are more preserved with the new method which is less diffusive.

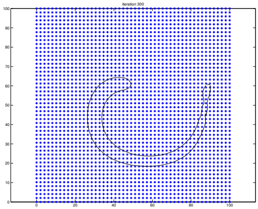
Disc in a vortex



Comparison with a Youngs reconstruction

- The figures below compares the deformation of the disc after 300 iterations (left : new IR method, right : Youngs) .





- The disc remains in one block with the new method, contrarily to the Youngs method case where a small bubble has already separated.

Conclusion

- A new interface reconstruction method that ensures volumic fractions preservation and continuity has been developped.
- It is based on a dynamic programming method to minimize volumic fractions errors, plus a correction step to put this error exactly to zero.
- The new method has been applied in various contexts, either static or coupled with an advection field.
- In all cases, it has shown its ability to reconstruct an accurate continuous interface that preserves volumic fractions, at a small computational cost.