



Operator-Splitting Approach for 3-T Radiation Diffusion in Two and Three Dimensions

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Acknowledgement:

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LA-UR-15-26559

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Outlines

- Motivation
- Interface reconstruction for radiation diffusion
- Coupled systems on 2D/3D general polyhedral meshes
- Operator splitting approaches
- Numerical examples
- Conclusions

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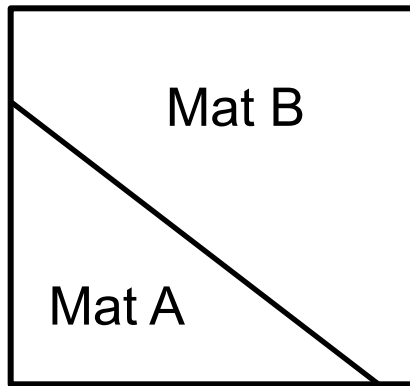
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- Developed for the program need.
- AMR, interface-aware hydro with strength in standard geometries.
- Non-equilibrium material pressure and temperature.
- Volume fraction material advection for sub-zonal physics models.
- Material strength models (Steinberg-Cochran-Guinan and PTW)
- Dynamical material mix models (Turbo and KL).
- Sesame and analytic EOS
- High explosives (DSD, Forest fire model, and reactive model).
- **Plasma 3-T radiation diffusion.**
- Basic resistive MHD with circuit for experiments.
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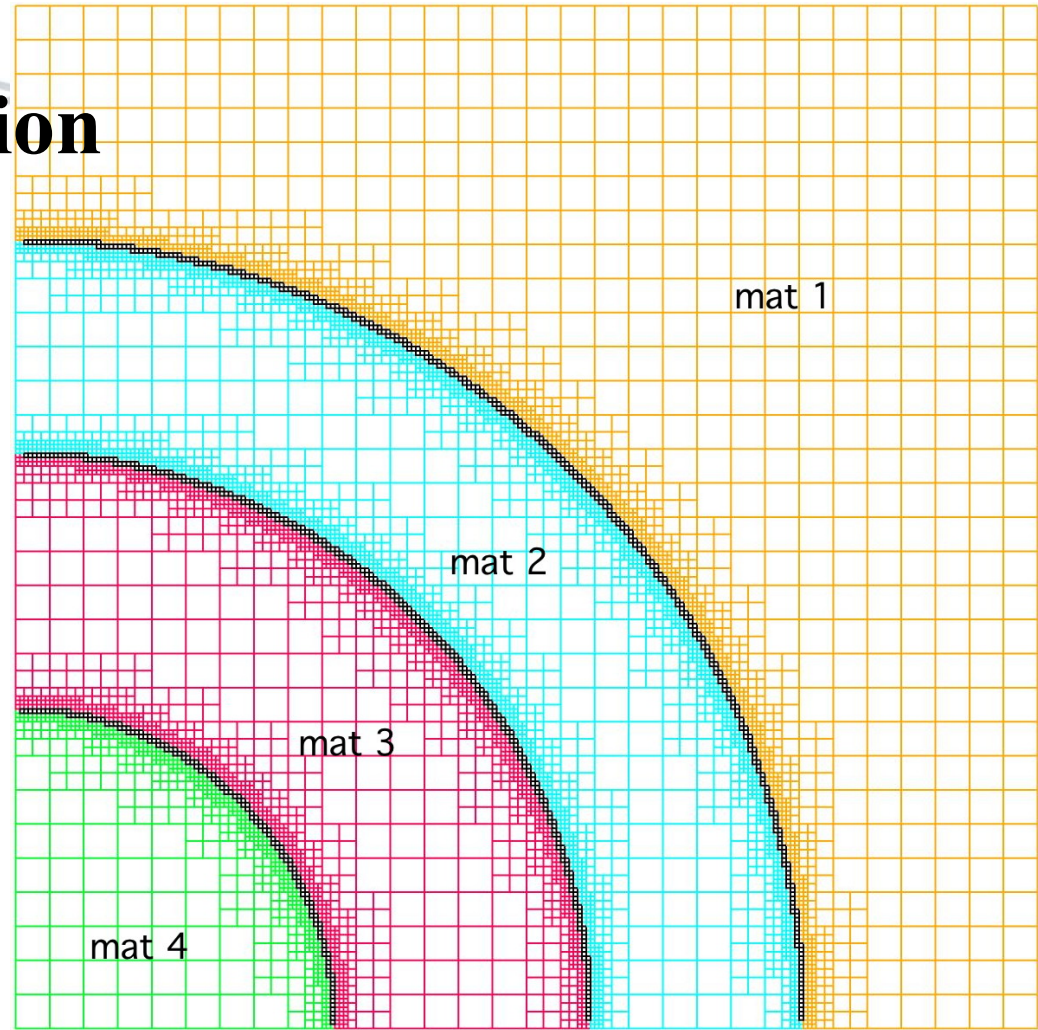


Interface reconstruction



Why ?

- non-equilibrium
- EOS unavailable for mixture of materials
- temperature-sensitive subsequent physics

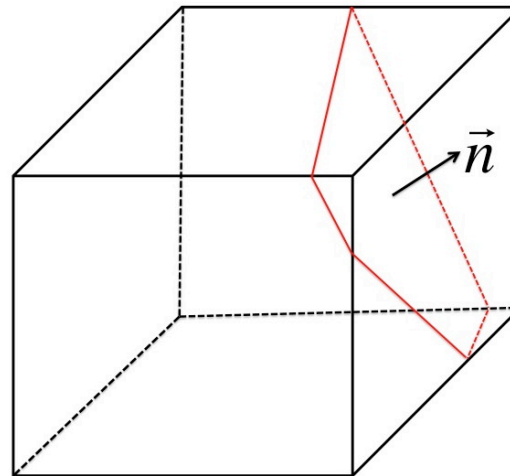
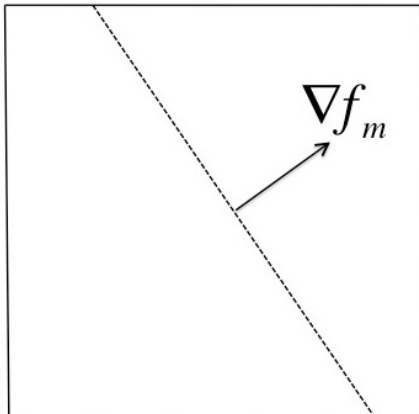


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Procedure of interface reconstruction

- Find gradient of each material within a mixed cell
- Determine the normal of the cell $q(m)$, only **one**
- Determine the order of materials $p(m)$
- Find interfaces to match ea volume of all mats in the cell



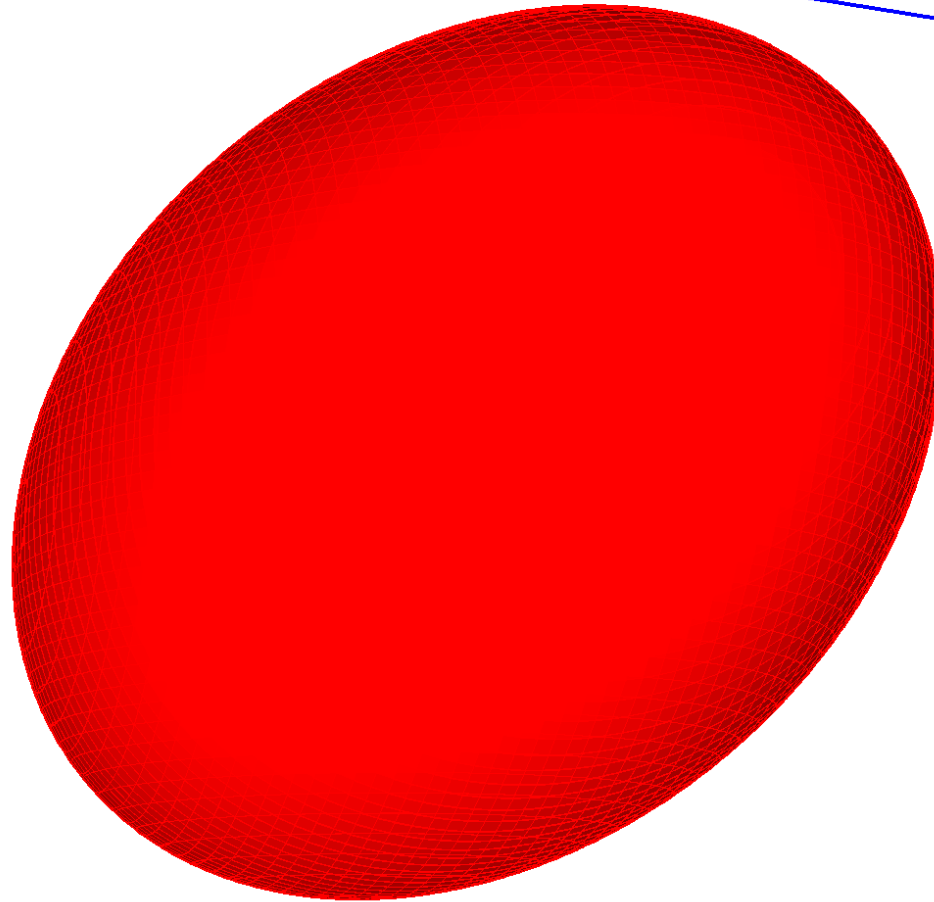
$$q(m) = |\nabla f_m|^2 \sqrt{f_m}$$

$$p(m) = n(m) \cdot n(m_0)$$

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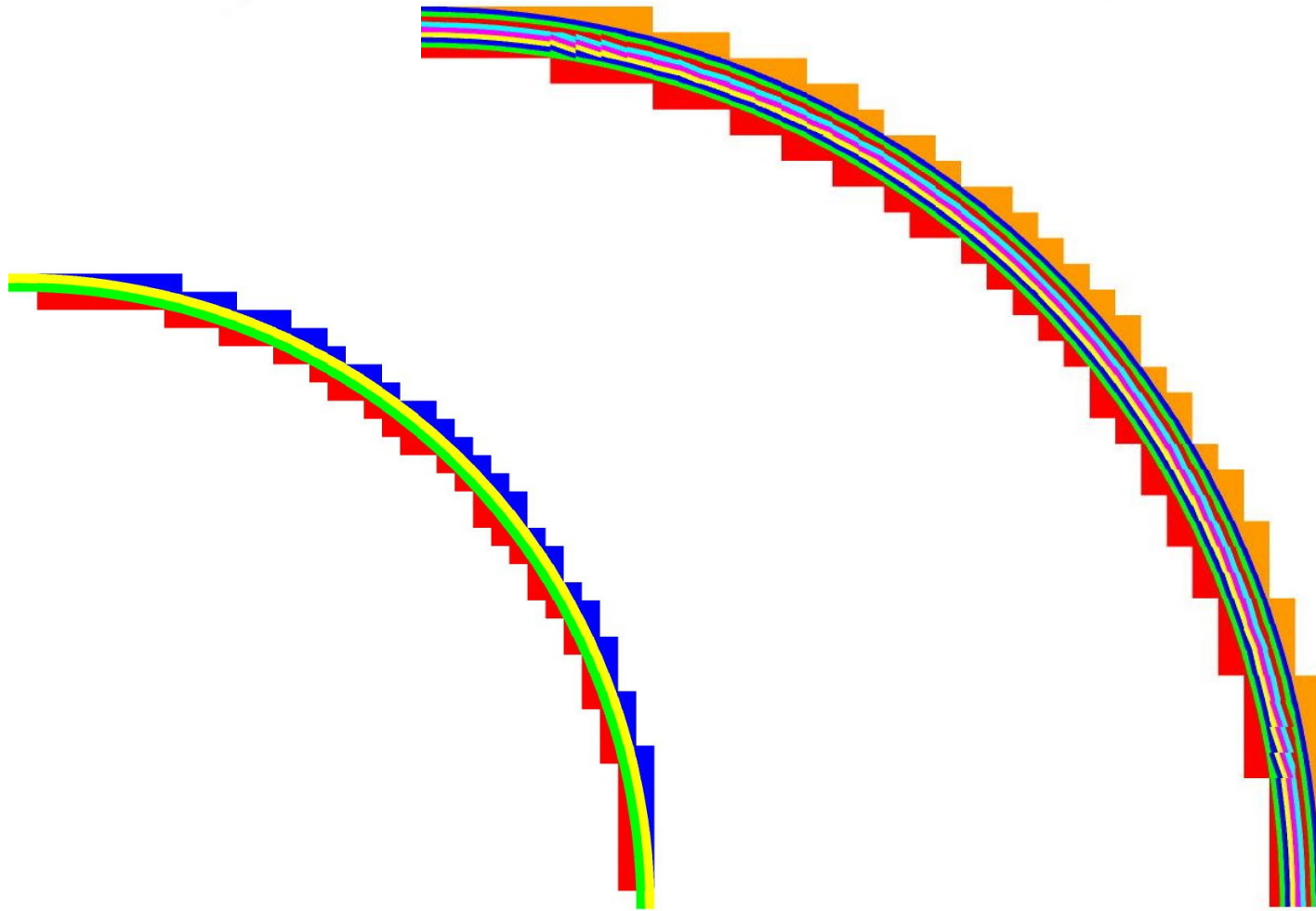


Reconstruction examples





Reconstruction examples

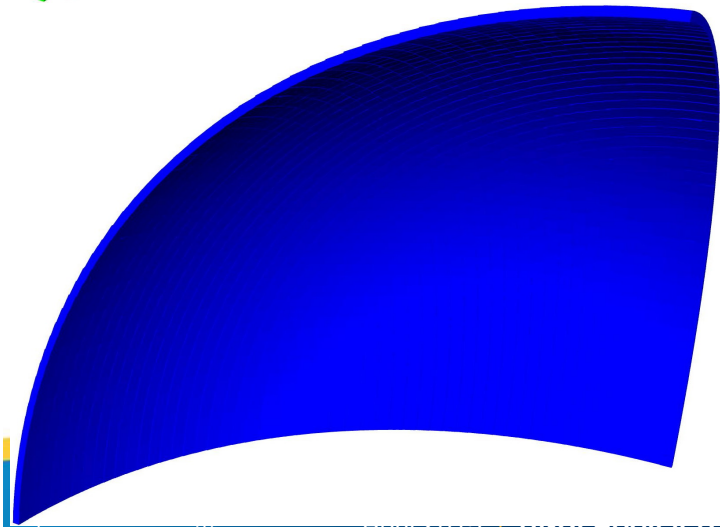
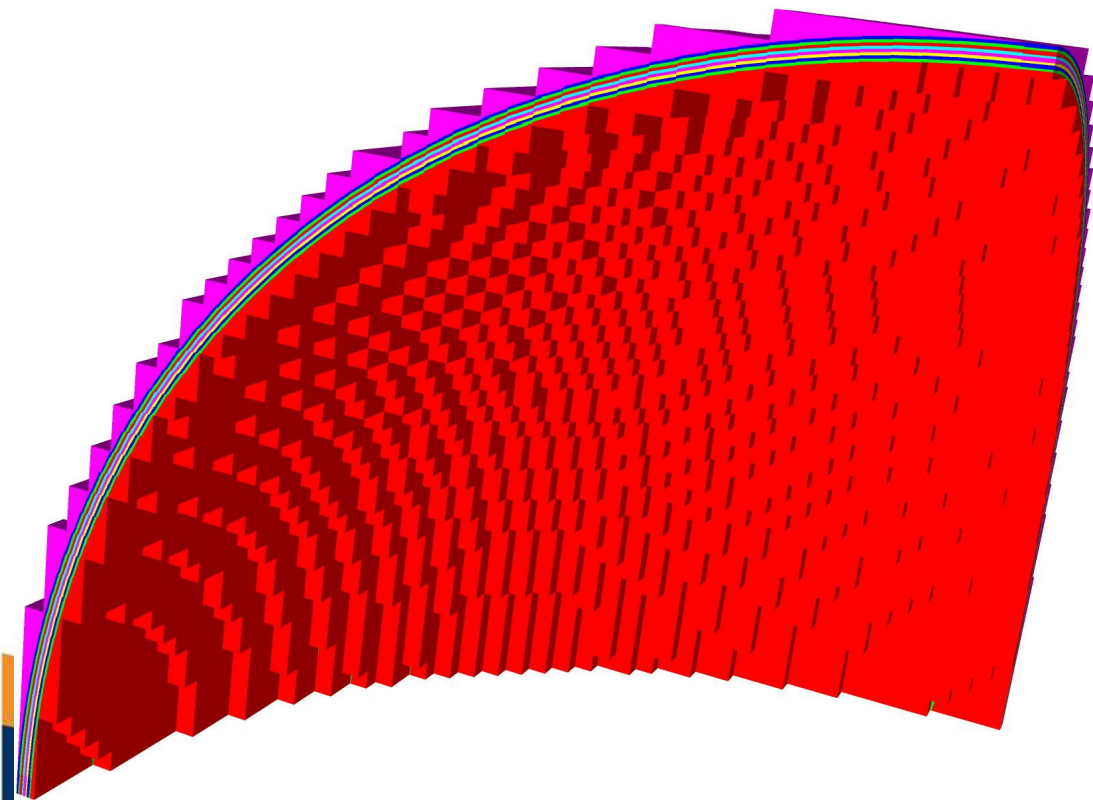
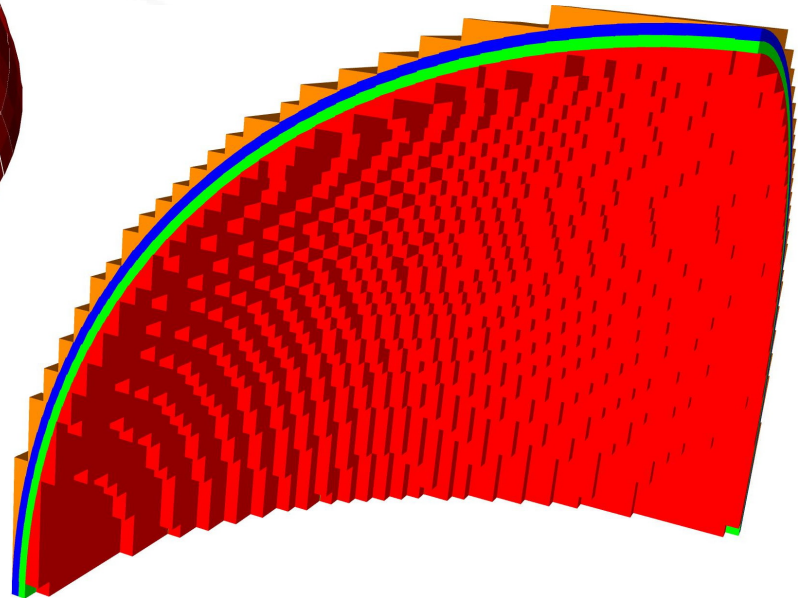
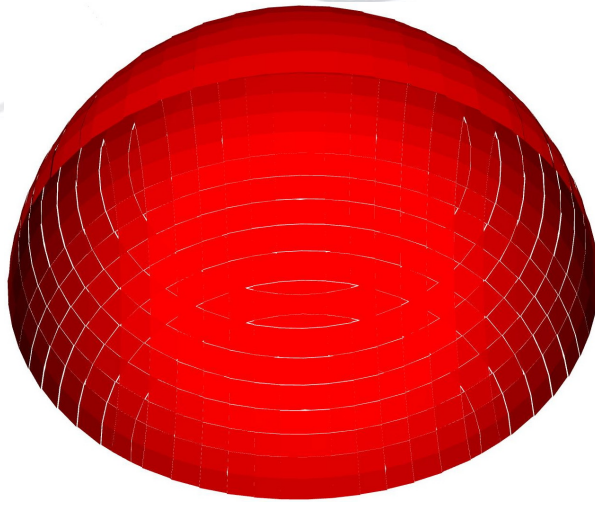
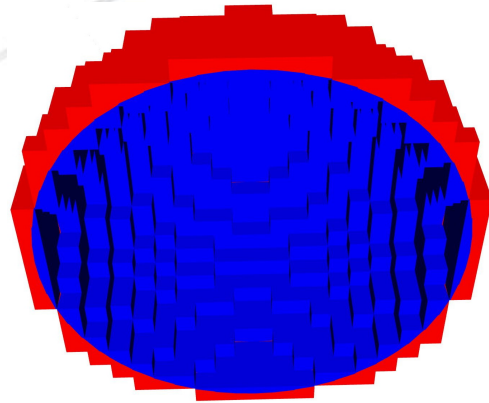


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Reconstruction examples





3-T radiation diffusion

$$\frac{\partial a T_r^4}{\partial t} = -\nabla \cdot \vec{F}_r + S_r,$$

$$\vec{F}_r \equiv -\sigma_r \nabla T_r^4,$$

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = -\nabla \cdot \vec{F}_e - S_r + S_e,$$

$$\vec{F}_e \equiv -\sigma_e \nabla T_e$$

$$\vec{F}_p \equiv -\sigma_p \nabla T_i$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\nabla \cdot \vec{F}_p - S_e.$$

$$S_r \equiv a c \rho \kappa_\rho (T_e^4 - T_r^4)$$

$$S_e \equiv \rho c_{ve} \kappa_{ie} (T_p - T_e)$$

a : radiation constant.

c_{ve}, c_{vp} : specific heat capacities.

$\sigma_r, \sigma_e, \sigma_p$: heat conductivities.

κ_ρ : material absorption coefficient.

κ_{pe} : coefficient for interaction.

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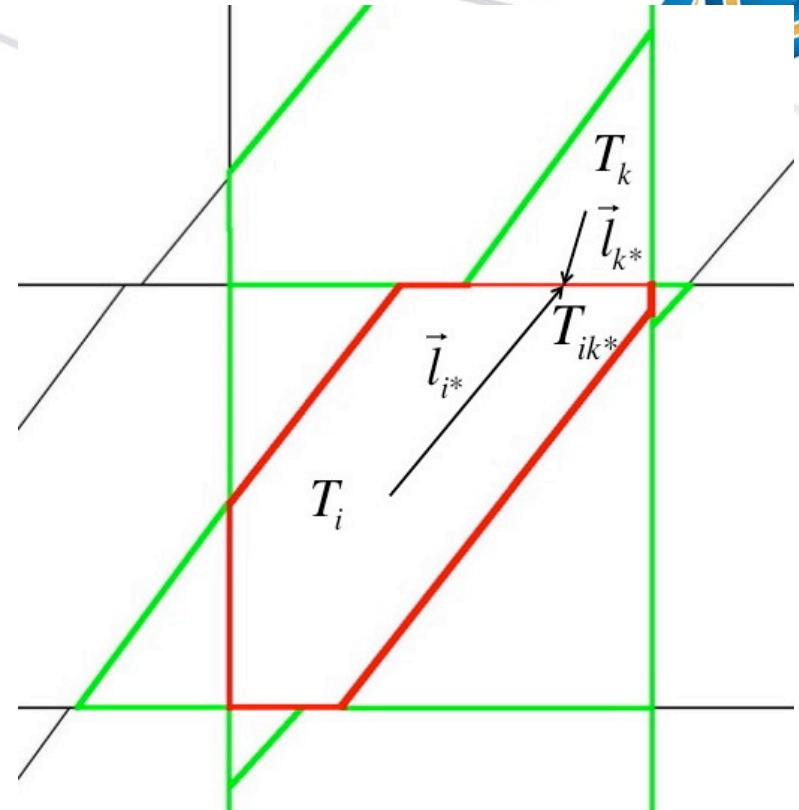
Different materials

Calculate $F_{ik}(T_i, T_k)$.

$$F_{ik} = -\sigma_{ik}(T_k - T_i)$$

$$T_{ik} = \frac{l_k \alpha_i \sigma_i T_i + l_i \alpha_k \sigma_k T_k}{l_k \alpha_i \sigma_i + l_i \alpha_k \sigma_k}$$

$$\sigma_{ik} = \frac{\sigma_i \sigma_k}{l_k \alpha_i \sigma_i + l_i \alpha_k \sigma_k} \alpha_i \alpha_k \sim \frac{\sigma}{l}$$



- W. Dai & P. Woodward, *Numerical simulations for nonlinear heat transfer in a system of multimaterial*, JCP **139** (1998)
- W. Dai & A. Scannapieco, *Second-order accurate interface- and discontinuity-aware diffusion solvers in two and three dimensions*, JCP **281** (2015).

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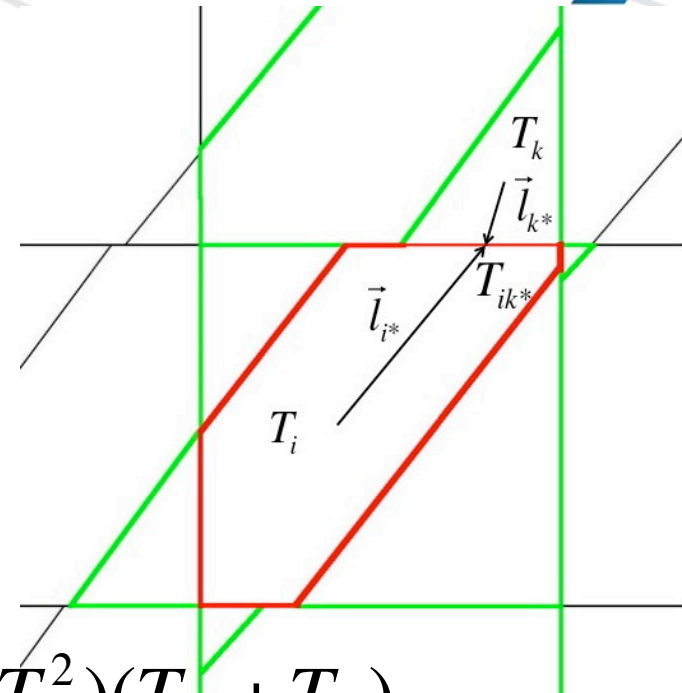
Radiation diffusion with different materials

Calculate $F_{ik}(T_i, T_k)$.

$$F_{ik} = -\sigma_{rik}(T_{rk} - T_{ri})$$

$$T_{rik}^4 = \frac{l_k \alpha_i \sigma_{ri} T_{ri}^4 + l_l \alpha_k \sigma_{rk} T_{rk}^4}{l_k \alpha_i \sigma_{ri} + l_l \alpha_k \sigma_{rk}}$$

$$\sigma_{rik} = \frac{\sigma_{ri} \sigma_{rk}}{l_k \alpha_i \sigma_{ri} + l_l \alpha_k \sigma_{rk}} \alpha_i \alpha_k (T_{rk}^2 + T_{ri}^2)(T_{rk} + T_{ri}).$$



- W. Dai & P. Woodward, JCP **139** (1998)
- W. Dai & A. Scannapieco, *Interface- and discontinuity-aware numerical schemes for plasma 3-T radiation diffusion in two and three dimensions*, JCP **300** (2015)

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Coupled nonlinear difference Eqs

$$a(T_r^4)_i^n + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{rik}^h A_{ik} \right) T_{ri}^h = a(T_r^4)_i + \frac{\Delta t}{\Delta V_i} \sum_k (\sigma_{rik}^h A_{ik} T_{rk}^h) + S_{ri}^h \Delta t$$

$$a(T_r^4)_i^h + \frac{3\Delta t}{4\Delta V_i} \left(\sum_k \sigma_{rik}^h A_{ik} \right) T_{ri}^h - \frac{\Delta t}{4\Delta V_i} \left(\sum_k \sigma_{rik}^n A_{ik} \right) T_{ri}^n =$$

$$a(T_r^4)_i + \frac{3\Delta t}{4\Delta V_i} \sum_k (\sigma_{rik}^h A_{ik} T_{rk}^h) - \frac{\Delta t}{4\Delta V_i} \sum_k (\sigma_{rik}^n A_{ik} T_{rk}^n) + \frac{1}{2} \Delta t \bar{S}_{ri}^h$$

$$(\rho c_e)_i T_{ei}^n + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{eik}^h A_{ik} \right) T_{ei}^h = (\rho c_e)_i T_{ei} + \frac{\Delta t}{\Delta V_i} \sum_k (\sigma_{eik}^h A_{ik} T_{ek}^h) - \Delta t (S_{ri}^h - S_{ei}^h)$$

$$(\rho c_e)_i T_{ei}^h + \frac{3\Delta t}{4\Delta V_i} \left(\sum_k \sigma_{eik}^h A_{ik} \right) T_{ei}^h - \frac{\Delta t}{4\Delta V_i} \left(\sum_k \sigma_{eik}^n A_{ik} \right) T_{ei}^n = (\rho c_e)_i T_{ei} + \frac{3\Delta t}{4\Delta V_i} \sum_k (\sigma_{eik}^h A_{ik} T_{ek}^h) - \frac{\Delta t}{4\Delta V_i} \sum_k (\sigma_{eik}^n A_{ik} T_{ek}^n) - \frac{1}{2} \Delta t (\bar{S}_{ri}^h - \bar{S}_{ei}^h)$$

$$(\rho c_p)_i T_{pi}^n + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{pik}^h A_{ik} \right) T_{pi}^h = (\rho c_p)_i T_{pi} + \frac{\Delta t}{\Delta V_i} \sum_k (\sigma_{pik}^h A_{ik} T_{pk}^h) - S_{pi}^h \Delta t$$

$$(\rho c_p)_i T_{pi}^h + \frac{3\Delta t}{4\Delta V_i} \left(\sum_k \sigma_{pik}^h A_{ik} \right) T_{pi}^h - \frac{\Delta t}{4\Delta V_i} \left(\sum_k \sigma_{pik}^n A_{ik} \right) T_{pi}^n = (\rho c_p)_i T_{pi} + \frac{3\Delta t}{4\Delta V_i} \sum_k (\sigma_{pik}^h A_{ik} T_{pk}^h) - \frac{\Delta t}{4\Delta V_i} \sum_k (\sigma_{pik}^n A_{ik} T_{pk}^n) - \frac{1}{2} \Delta t \bar{S}_{pi}^h$$

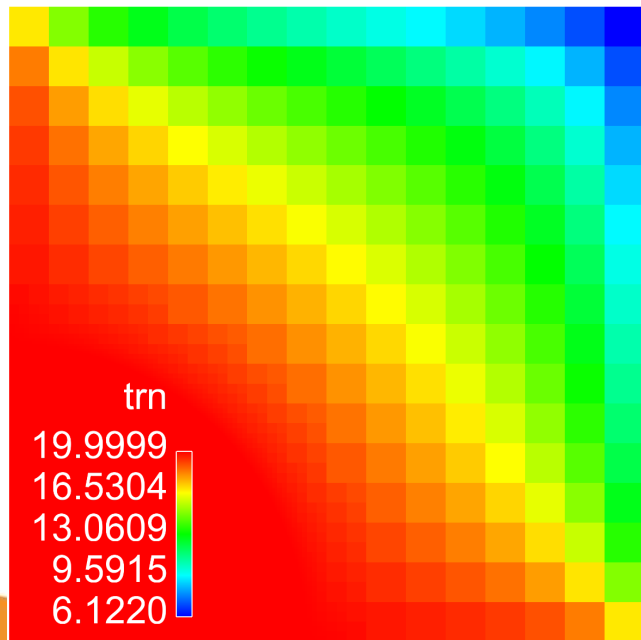
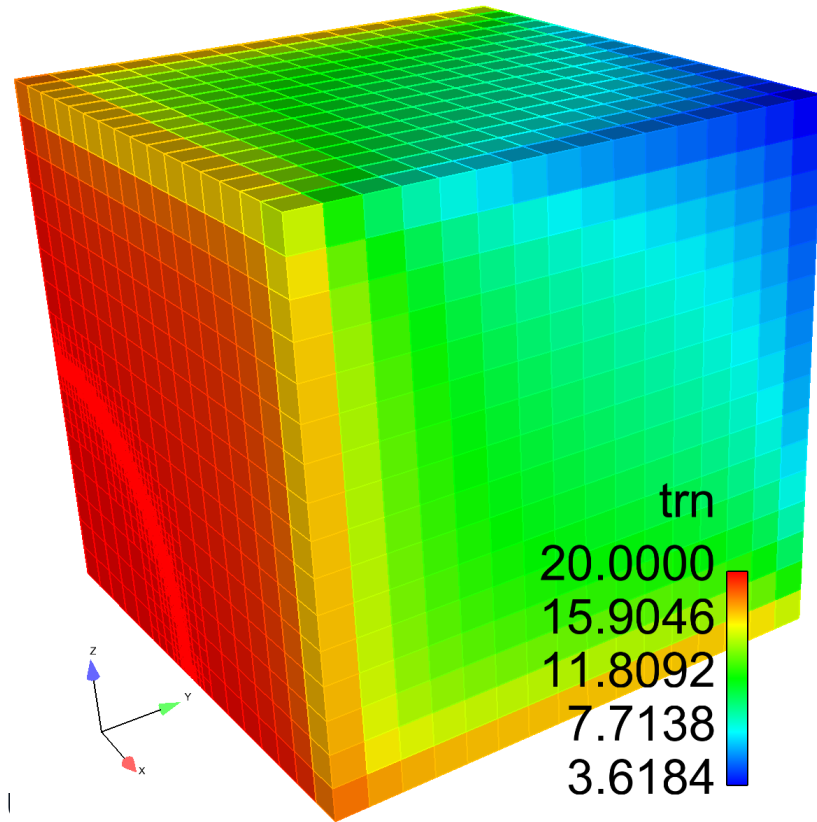
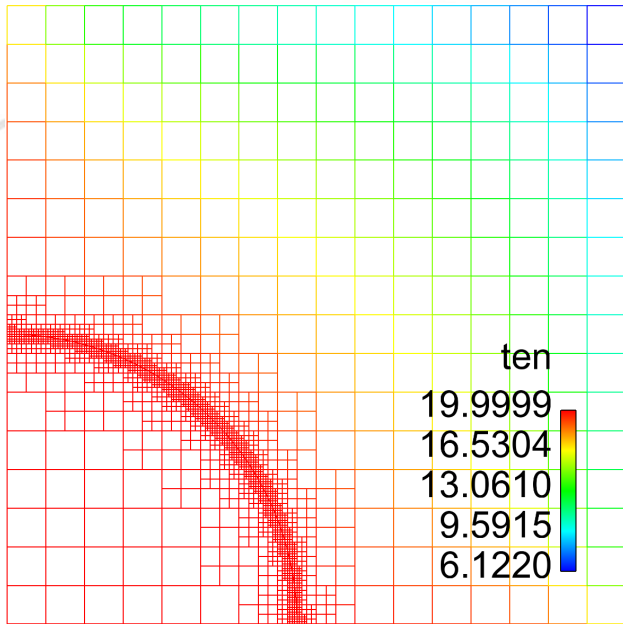
$$\bar{S}_i^h = \frac{3}{2} S_i^h - \frac{1}{2} S_i^n$$

$$S_{ri}^h = ac\rho\kappa_\rho (T_{ei}^{h4} - T_{ri}^{h4}). \quad S_{ri}^n = ac\rho\kappa_\rho (T_{ei}^{n4} - T_{ri}^{n4}).$$

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Example of steady state of 3-T after one large time step





1st order accuracy in time, fully nonlinear, implicit & coupled

$$a(T_r^4)_i^n + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{rik}^n A_{ik} \right) T_{ri}^n = a(T_r^4)_i + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{rik}^n A_{ik} \right) T_{rk}^n + S_{ri}^n \Delta t$$

$$(\rho c_e)_i T_{ei}^n + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{eik}^n A_{ik} \right) T_{ei}^n = (\rho c_e)_i T_{ei} + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{eik}^n A_{ik} \right) T_{ek}^n - \Delta t (S_{ri}^n - S_{ei}^n)$$

$$(\rho c_p)_i T_{pi}^n + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{pik}^n A_{ik} \right) T_{pi}^n = (\rho c_p)_i T_{pi} + \frac{\Delta t}{\Delta V_i} \left(\sum_k \sigma_{pik}^n A_{ik} \right) T_{pk}^n - S_{pi}^n \Delta t$$

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Simple but inappropriate split algorithm

Step1)

$$\begin{aligned}\frac{\partial aT_r^4}{\partial t} &= -\nabla \cdot (\sigma_r \nabla T_r), \\ C_{ve} \frac{\partial T_e}{\partial t} &= -\nabla \cdot (\sigma_e \nabla T_e), \\ C_{vp} \frac{\partial T_p}{\partial t} &= -\nabla \cdot (\sigma_p \nabla T_p).\end{aligned}$$

Step2)

$$\begin{aligned}\frac{\partial aT_r^4}{\partial t} &= ac\rho\kappa_p(T_e^4 - T_r^4), \\ C_{ve} \frac{\partial T_e}{\partial t} &= -ac\rho\kappa_p(T_e^4 - T_r^4) + C_{ve}\kappa_{ie}(T_p - T_e), \\ C_{vp} \frac{\partial T_p}{\partial t} &= -C_{ve}\kappa_{ie}(T_p - T_e).\end{aligned}$$

All are fully implicitly solved.

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Recommended operator-splitting

Step 1

$$a \frac{\partial T_r^4}{\partial t} = -\nabla \cdot \vec{F}_r + S_r$$

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = -S_r + \frac{1}{3} S_e$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\frac{1}{3} S_e$$

Step 2

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = -\nabla \cdot \vec{F}_e + \frac{1}{3} S_e$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\frac{1}{3} S_e$$

Step 3

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = \frac{1}{3} S_e$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\nabla \cdot \vec{F}_p - \frac{1}{3} S_e$$

Each step is implicitly solved, including source.

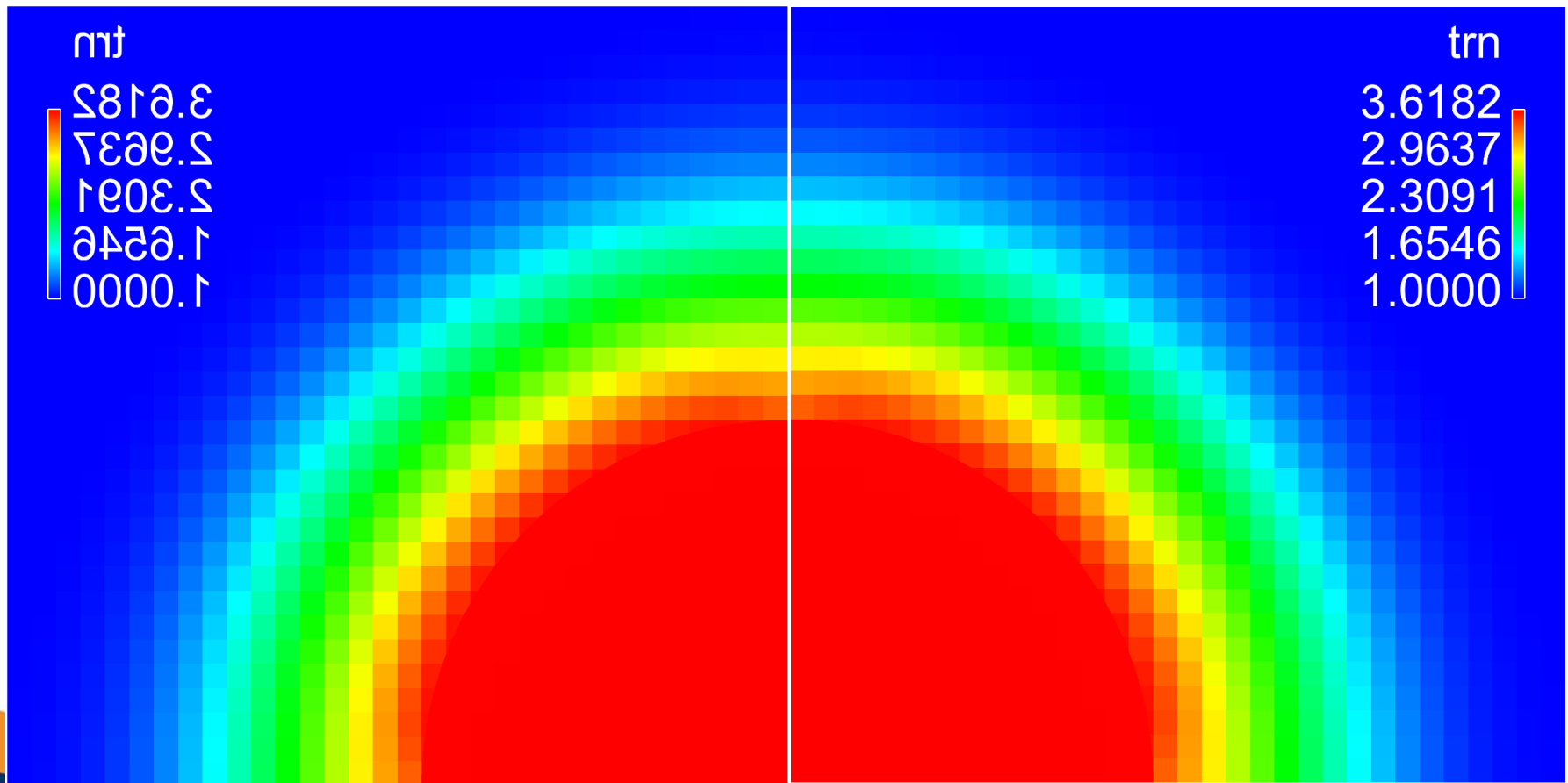
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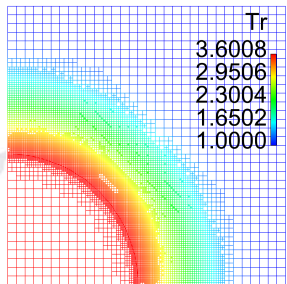


Numerical tests:

Radiation diffusion only: 10 time steps of nonlinear & splitting

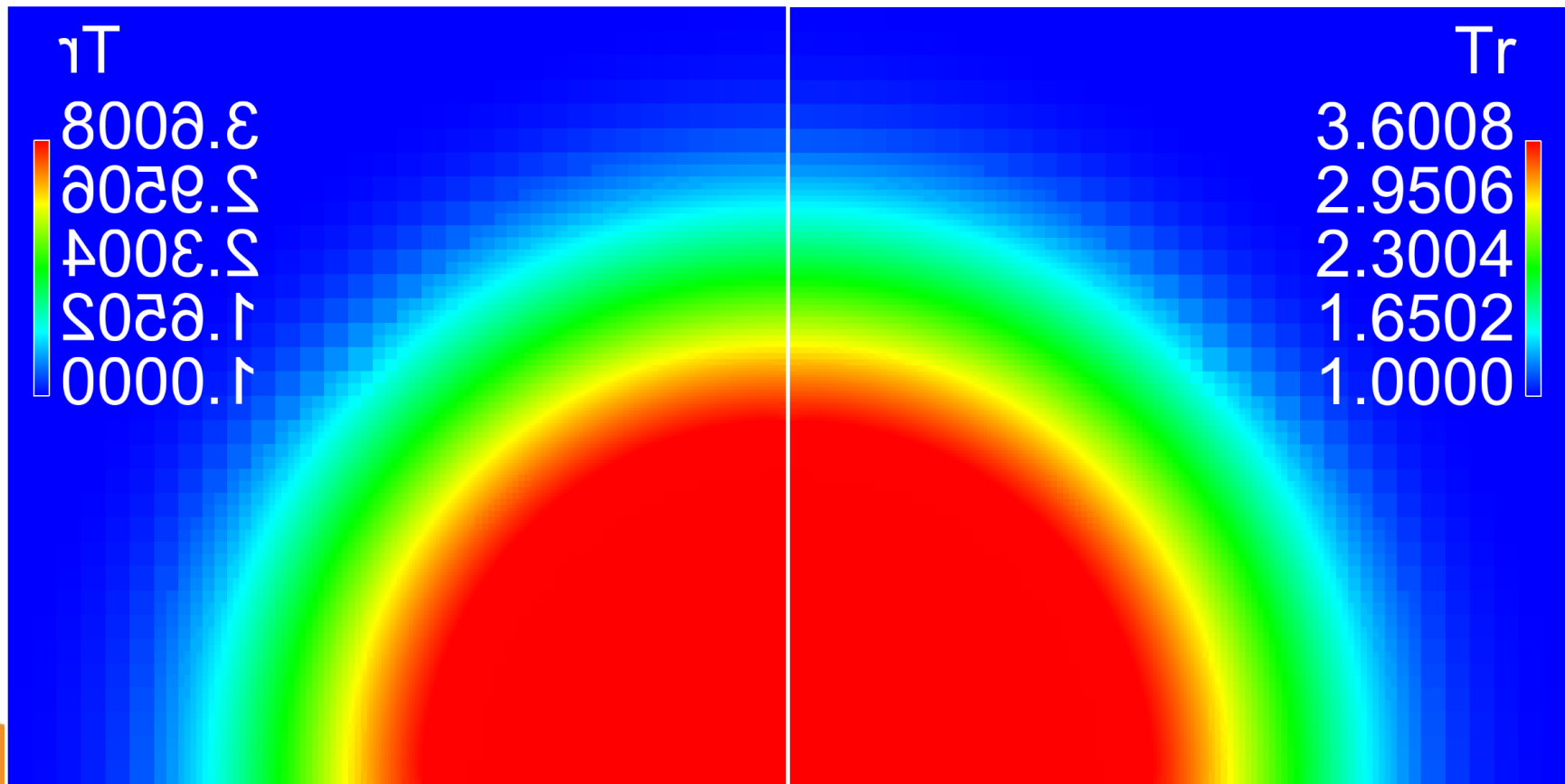
Reconstructed on uniform mesh





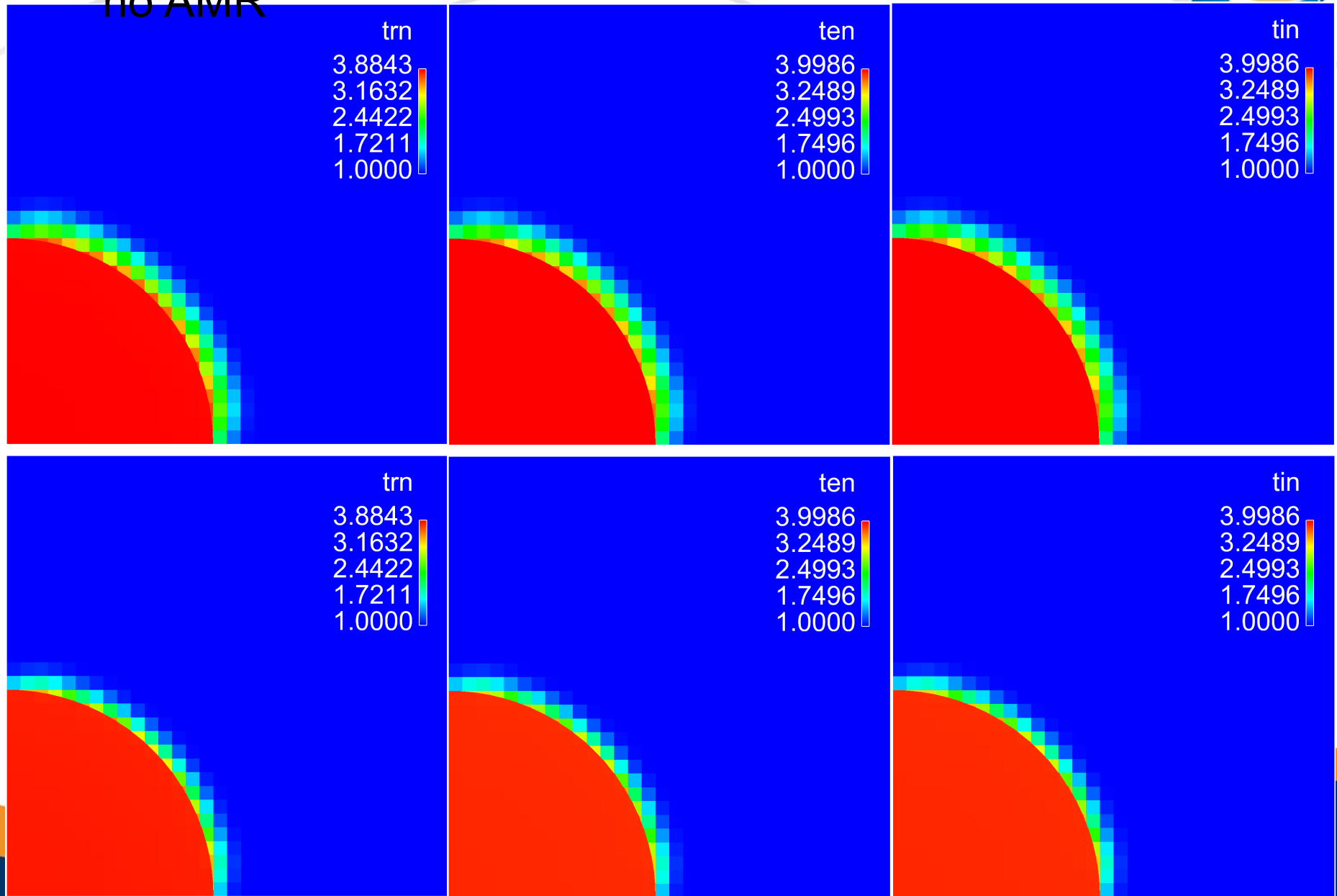
Radiation diffusion only: 10 time steps of nonlinear & splitting

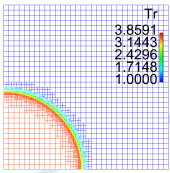
Reconstructed on AMR mesh



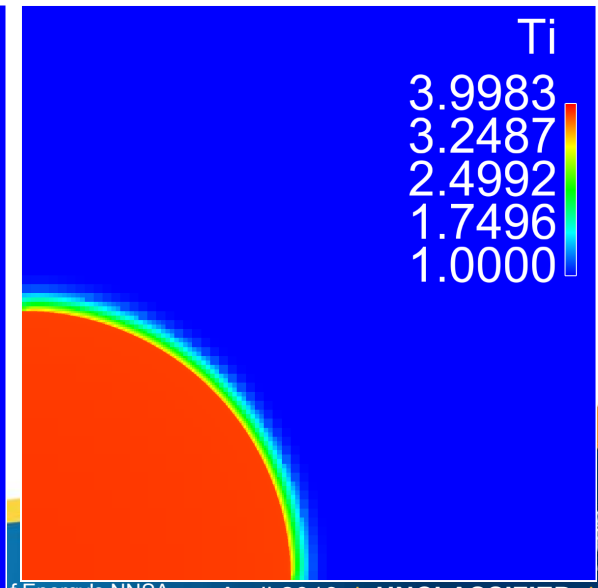
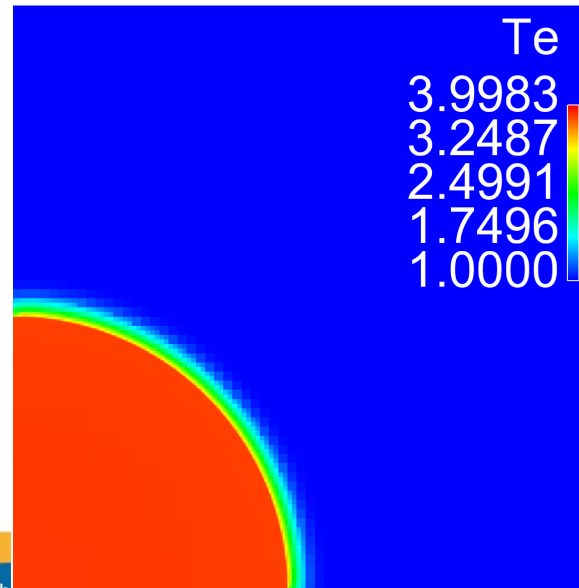
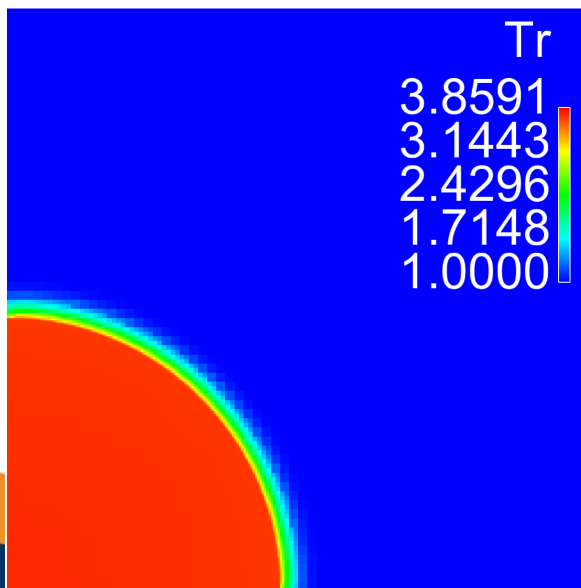
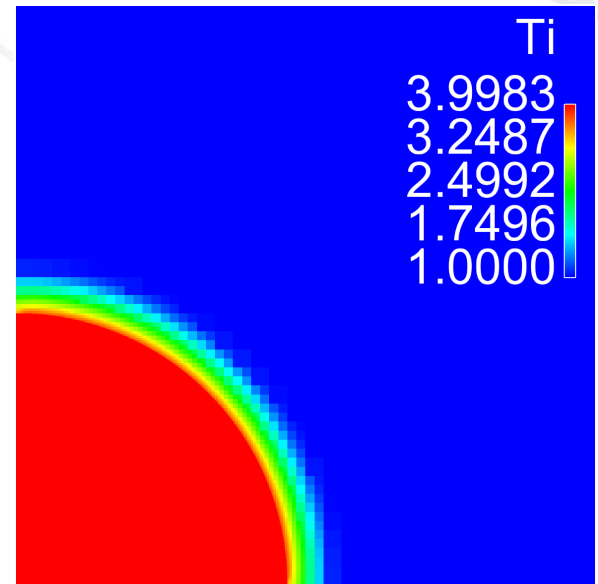
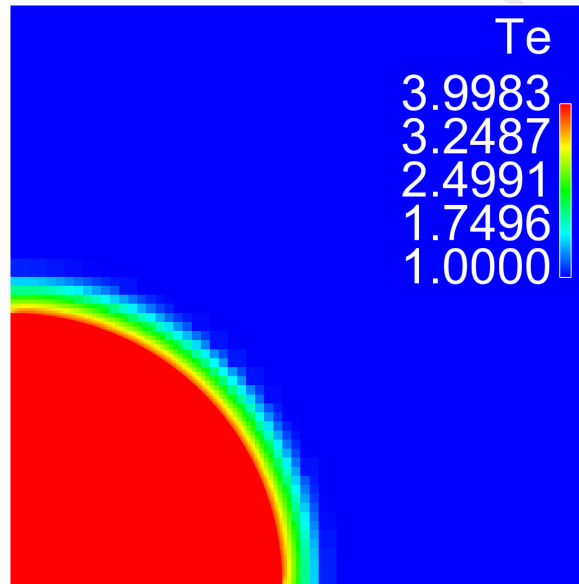
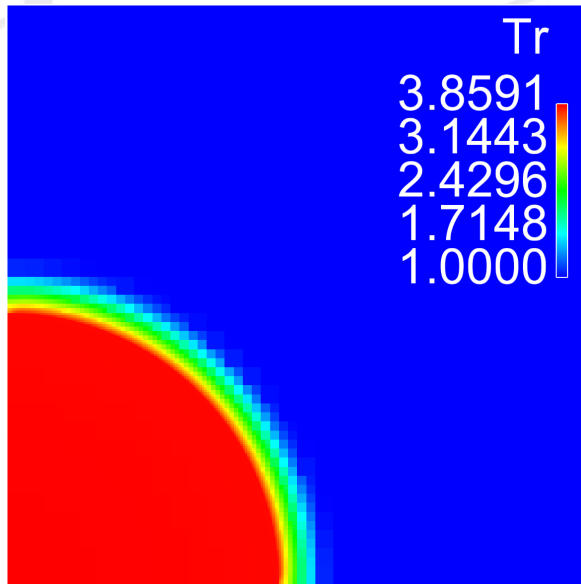


3T: Nonlinear & splitting after a relatively large time step, no AMR



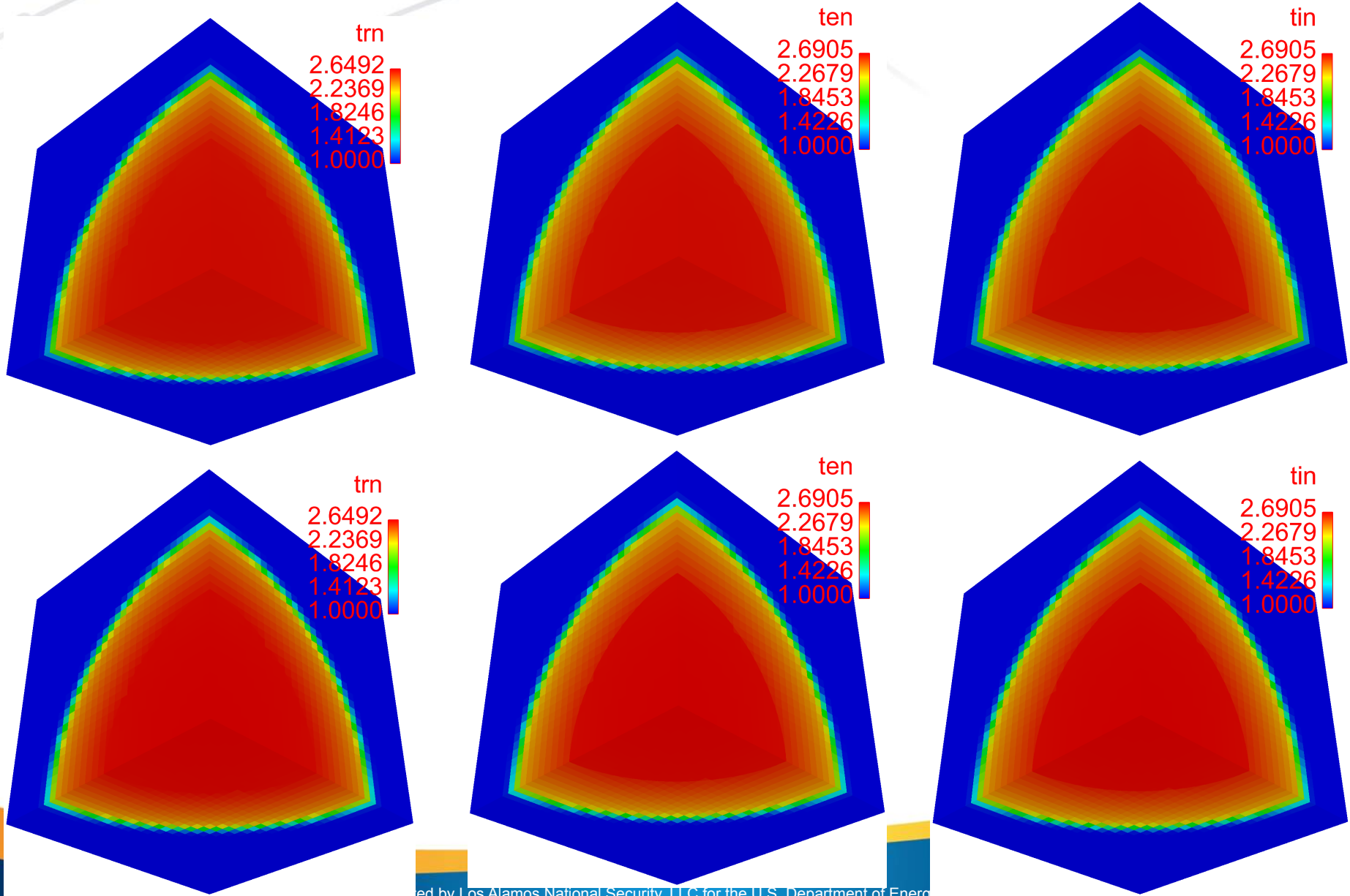


3T: 1 time step of nonlinear & splitting, with AMR



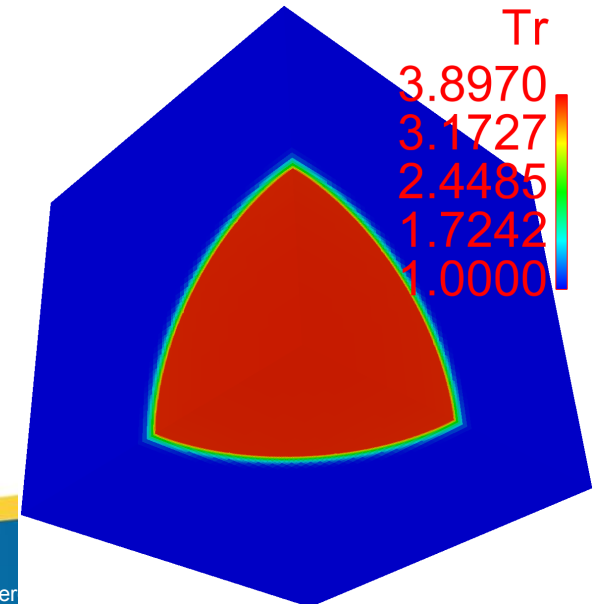
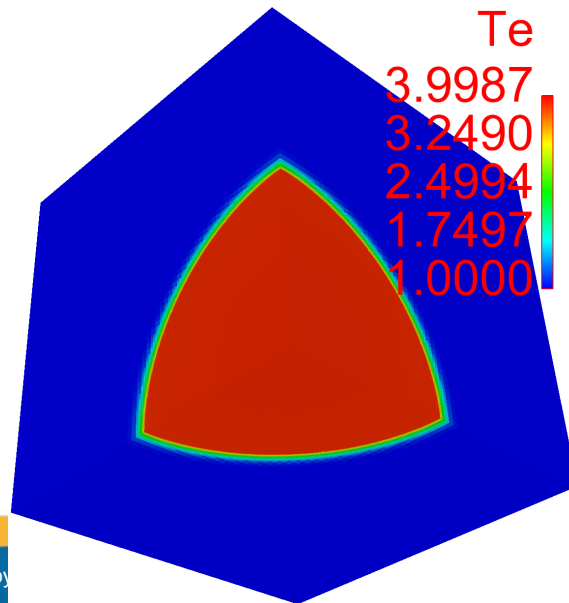
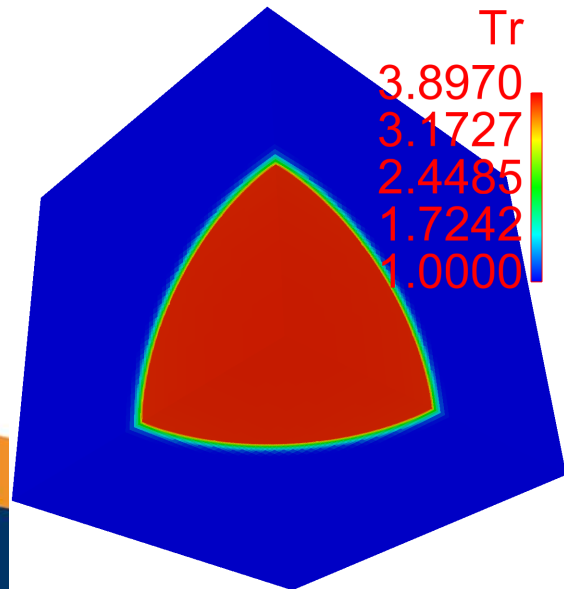
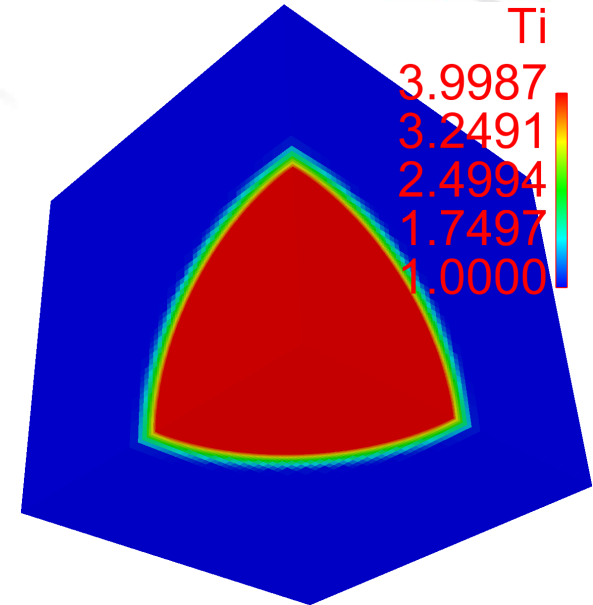
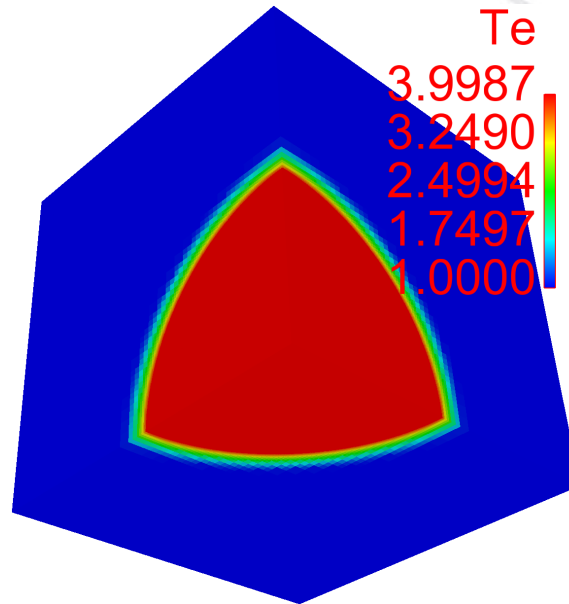
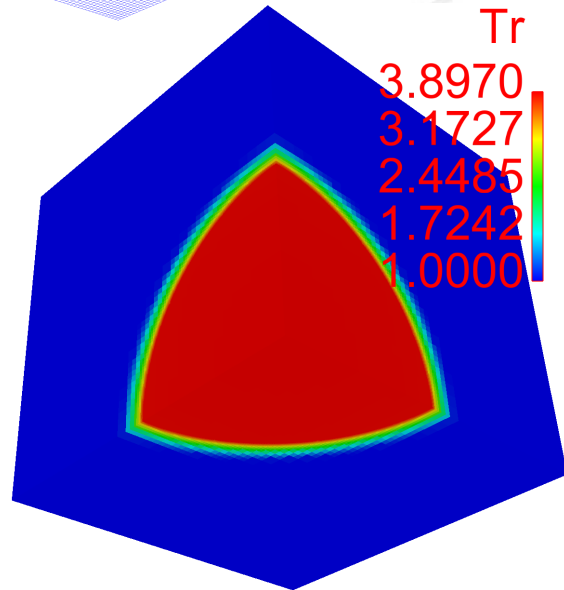
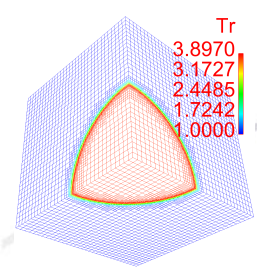


3T: 10 time steps of nonlinear & splitting, no AMR



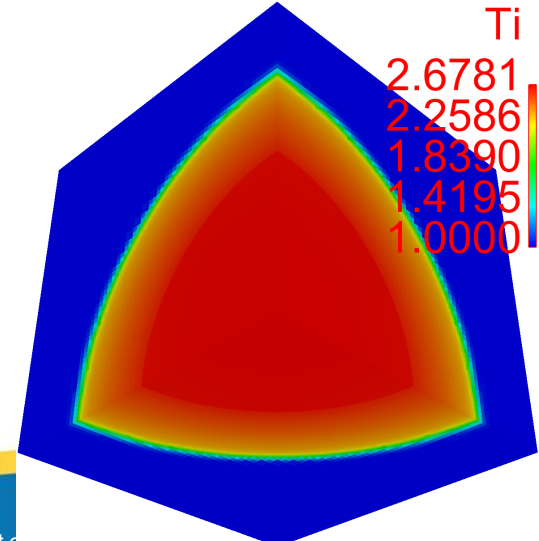
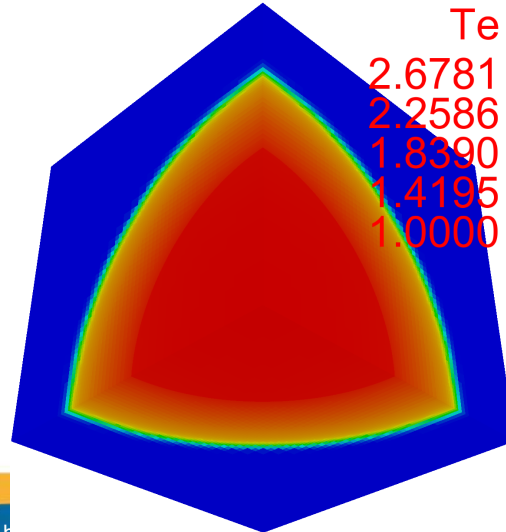
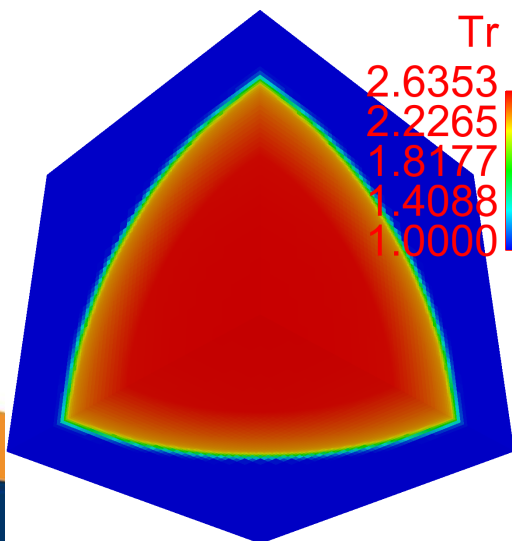
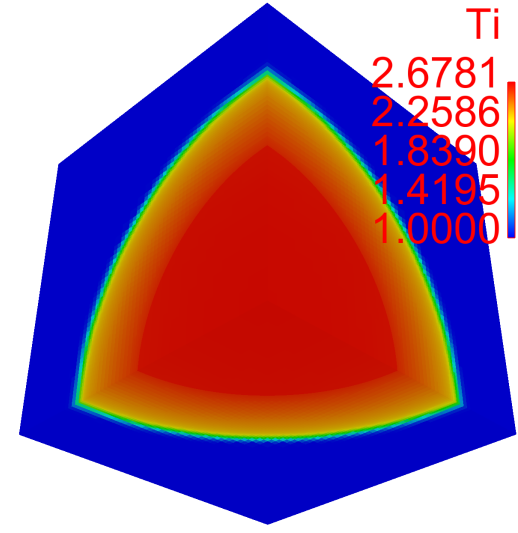
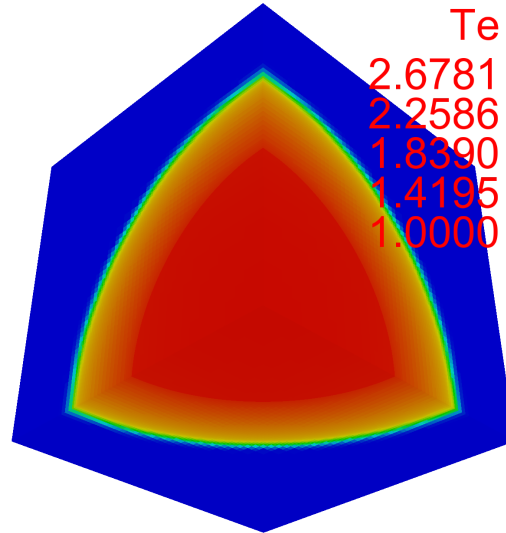
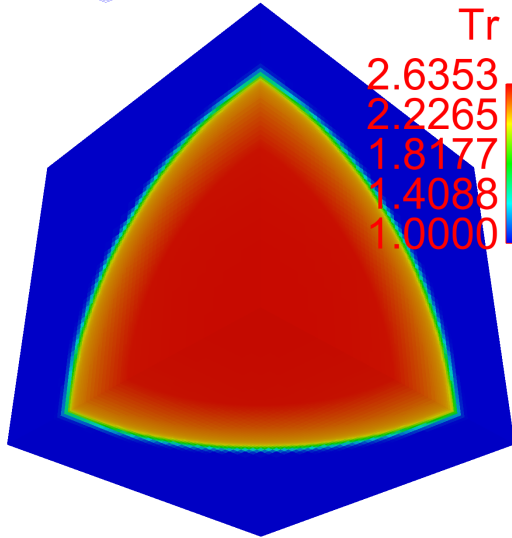
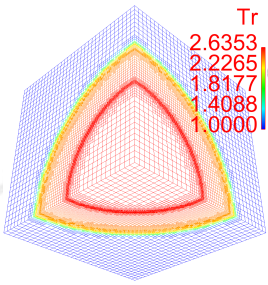


3T: 1 time step of nonlinear & splitting, AMR





3T: 10 time steps of linear & splitting, AMR





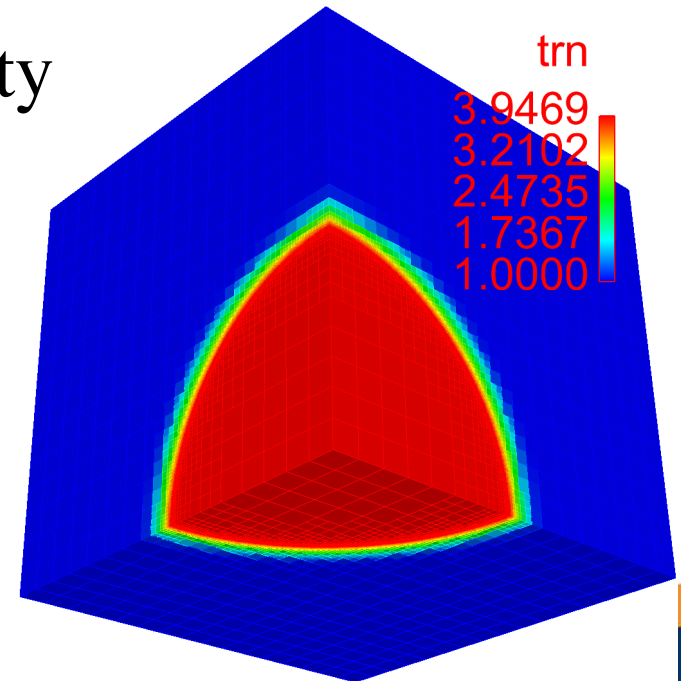
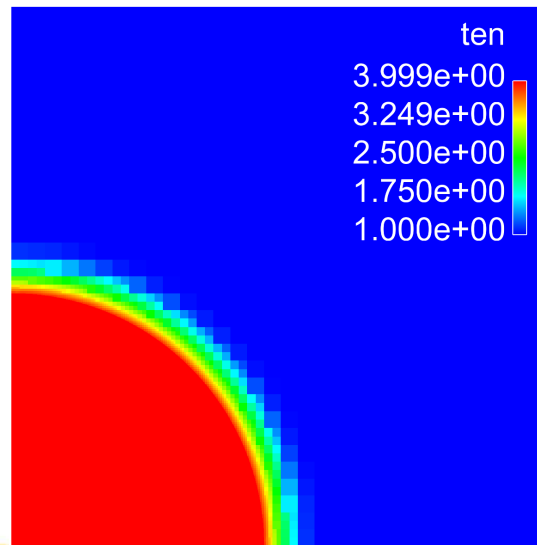
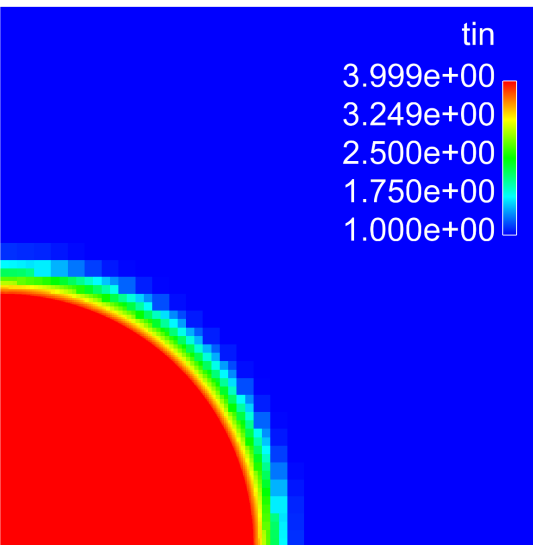
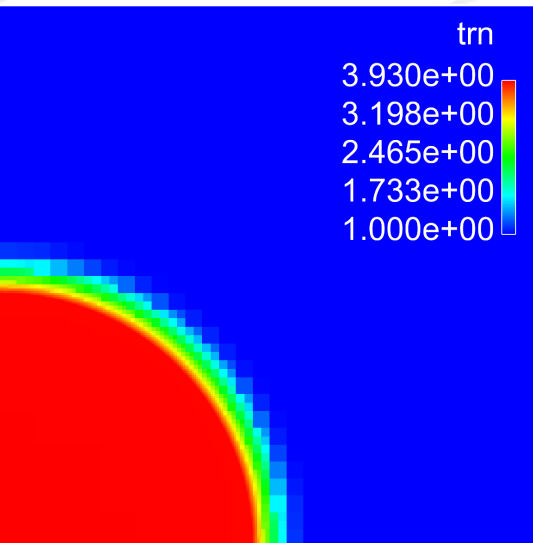
Solution after a relatively large time step, AMR

Fully nonlinear, fully coupled, fully implicit

large σ_r , small σ_e , σ_p

strong coupling

large heat capacity





Inappropriate split algorithm for this problem

Step1)

$$\frac{\partial aT_r^4}{\partial t} = -\nabla \cdot (\sigma_r \nabla T_r),$$

$$C_{ve} \frac{\partial T_e}{\partial t} = -\nabla \cdot (\sigma_e \nabla T_e),$$

$$C_{vp} \frac{\partial T_p}{\partial t} = -\nabla \cdot (\sigma_p \nabla T_p).$$

all fully
Implicitly
Solved.


Step2)

$$\frac{\partial aT_r^4}{\partial t} = ac\rho\kappa_p(T_e^4 - T_r^4),$$


$$C_{ve} \frac{\partial T_e}{\partial t} = -ac\rho\kappa_p(T_e^4 - T_r^4) + C_{ve}\kappa_{ie}(T_p - T_e),$$

$$C_{vp} \frac{\partial T_p}{\partial t} = -C_{ve}\kappa_{ie}(T_p - T_e).$$


trn
3.9942e+00
3.2457e+00
2.4971e+00
1.7486e+00
1.0000e+00



ten
3.996e+00
3.247e+00
2.498e+00
1.749e+00
1.000e+00



tin
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Tr diffused, but absorbed

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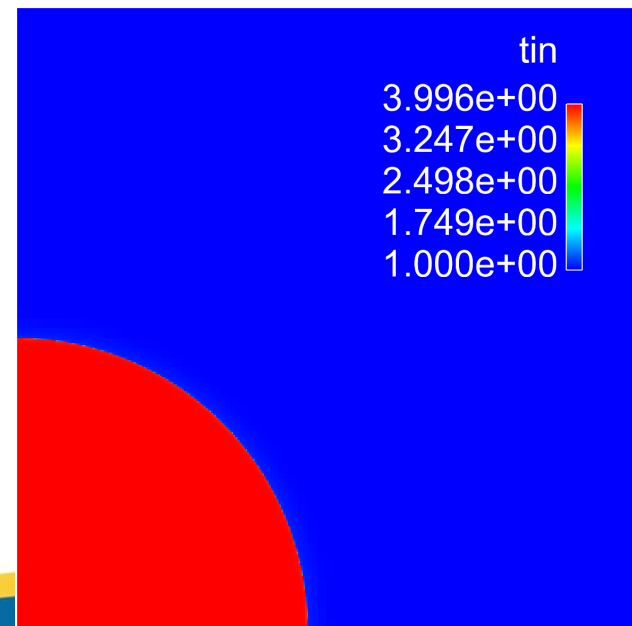
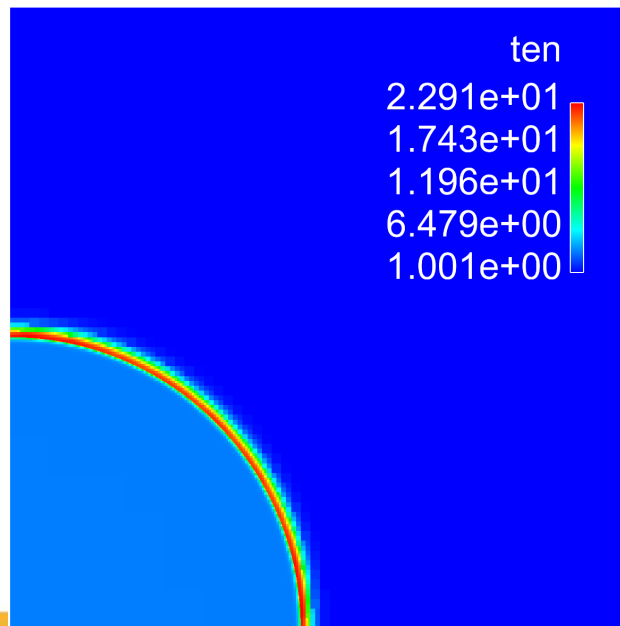
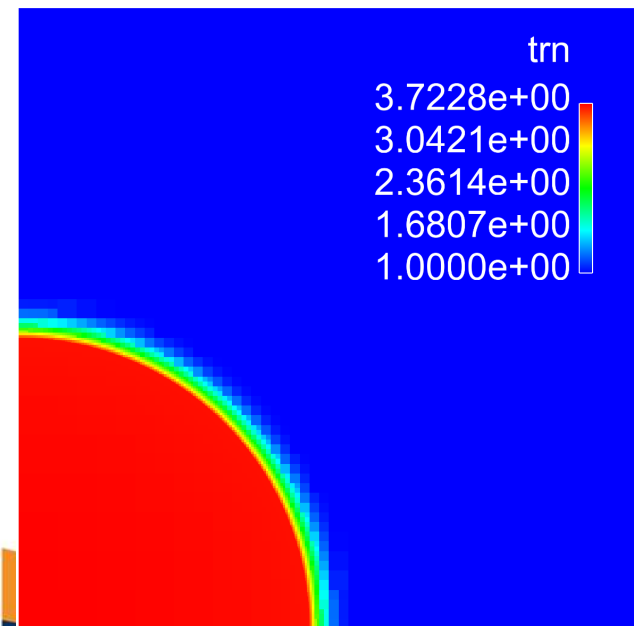
Another inappropriate splitting algorithm

$$\begin{aligned}\frac{\partial aT_r^4}{\partial t} &= -\nabla \cdot \vec{F}_r + S_r, \\ C_{ve} \frac{\partial T_e}{\partial t} &= -S_r + \frac{2}{3} S_e, \\ C_{vp} \frac{\partial T_p}{\partial t} &= -\frac{2}{3} S_e.\end{aligned}$$

$$\begin{aligned}C_{ve} \frac{\partial T_e}{\partial t} &= -\nabla \cdot \vec{F}_e + \frac{1}{3} S_e, \\ C_{vp} \frac{\partial T_p}{\partial t} &= -\frac{1}{3} S_e.\end{aligned}$$

$$C_{vp} \frac{\partial T_p}{\partial t} = -\nabla \cdot \vec{F}_p.$$

$$(T_r^4 - T_e^4)^n \approx (T_r^4 - T_e^4) + 4(T_e^3 T_e^n - T_r^3 T_r^n)$$





Fully implicit, but linear and splitting

$$a \frac{\partial T_r^4}{\partial t} = -\nabla \cdot \vec{F}_r + S_r$$

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = -S_r + \frac{1}{3} S_e$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\frac{1}{3} S_e$$

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = -\nabla \cdot \vec{F}_e + \frac{1}{3} S_e$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\frac{1}{3} S_e$$

$$\rho c_{ve} \frac{\partial T_e}{\partial t} = \frac{1}{3} S_e$$

$$\rho c_{vp} \frac{\partial T_p}{\partial t} = -\nabla \cdot \vec{F}_p - \frac{1}{3} S_e$$

trn

3.840e+00
3.130e+00
2.420e+00
1.710e+00
1.000e+00

ten

3.904e+00
3.178e+00
2.452e+00
1.726e+00
1.000e+00

tin

3.907e+00
3.180e+00
2.453e+00
1.727e+00
1.000e+00



Conclusions

We proposed one splitting approach for 3-T radiation diffusion. We compared the solutions with those of fully nonlinear and fully coupled systems and those of coupled and linear systems. The solutions of the splitting approach work quite well for small time steps, and reasonably well for large time step. We also pointed out the drawbacks of some existing splitting approaches.

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References

- W. Dai, P.R. Woodward, *Numerical simulations for nonlinear heat transfer in a system of multimaterials*, JCP, **139** (1998), pp. 58–78.
- W. Dai, P.R. Woodward, *Numerical simulations for radiation hydrodynamics II. Transport limit*, JCP, **157** (2000), pp. 199–233.
- W. Dai, A.J. Scannapieco, *Second-order accurate interface- and discontinuity-aware diffusion solvers in two and three dimensions*, J. Comput. Phys., **281** (2015), pp. 982–1002.
- W. Dai, A.J. Scannapieco, *Interface- and discontinuity-aware numerical schemes for plasma 3-T radiation diffusion in two and three dimensions*, JCP, **300** (2015), pp. 643–664.

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backup

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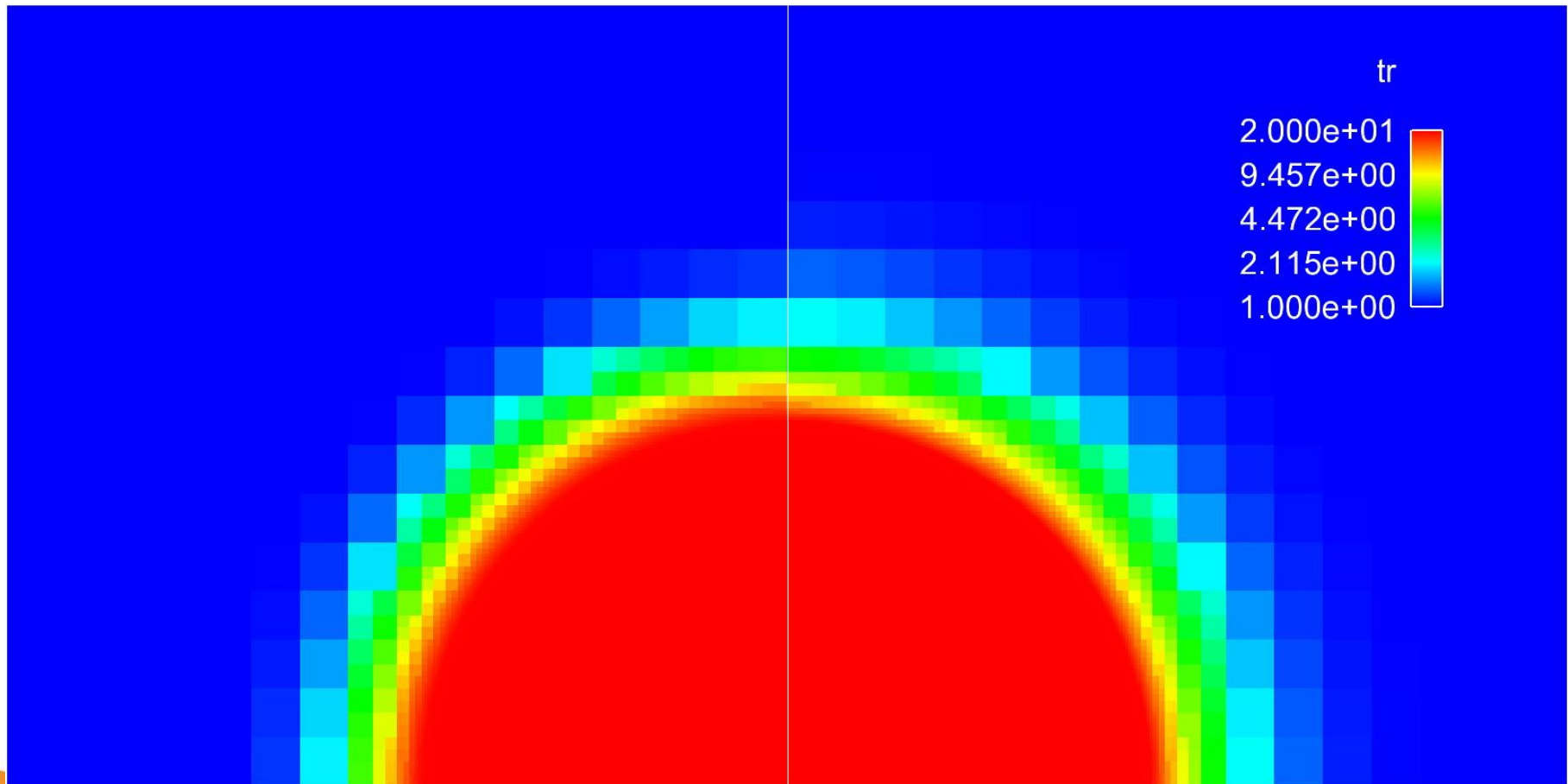




1nd vs 2nd order accuracy in time

2nd order in time

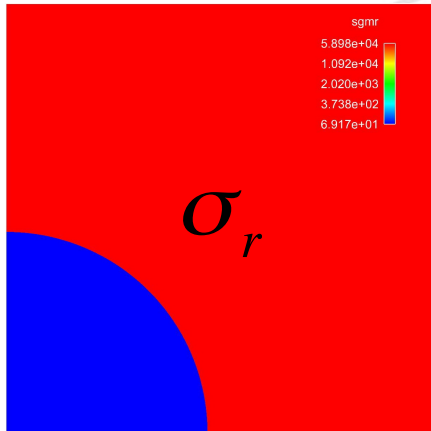
1st order in time



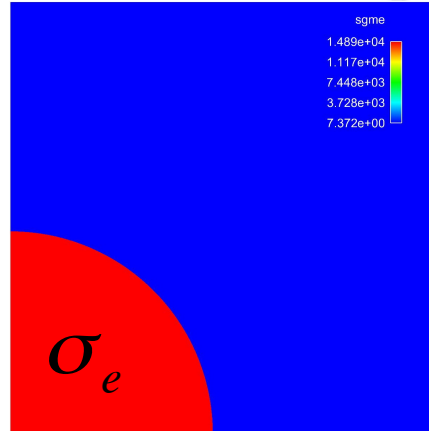
After one time step $\Delta t = 10^{-4} \mu s$



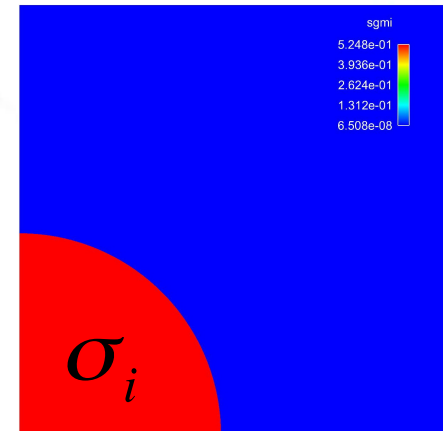
3-T and fully coupling



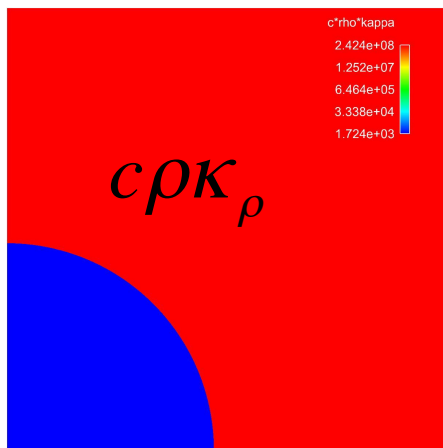
$69 \sim 6 \times 10^4$



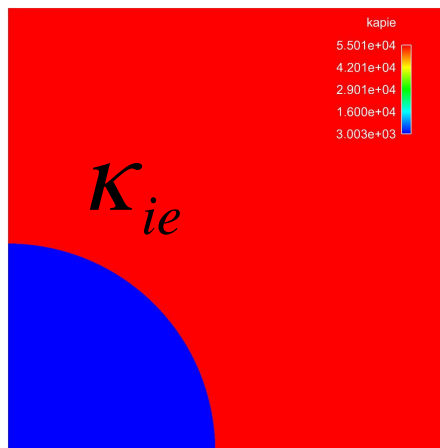
$7.4 \sim 1.5 \times 10^4$



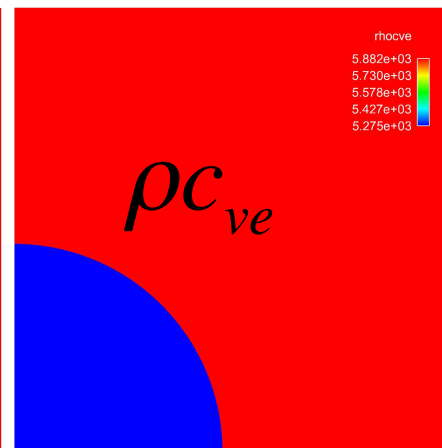
$6 \times 10^{-8} \sim 0.5$



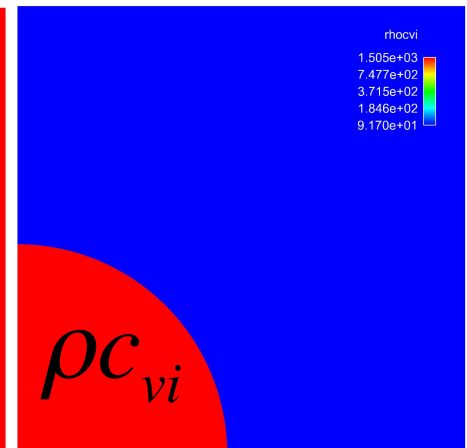
$1.7 \times 10^3 \sim 2.4 \times 10^8$



$3.0 \times 10^3 \sim 5.5 \times 10^4$

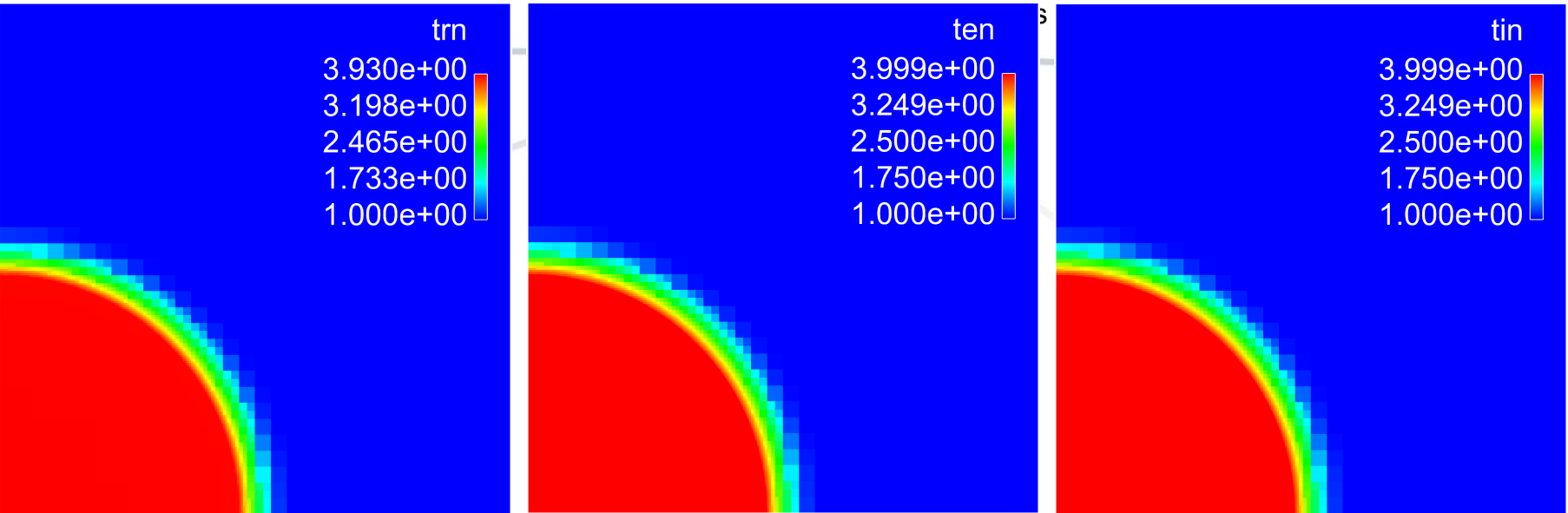


5275 ~ 5882



91 ~ 1505

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Above: fully implicit, fully coupled, and fully nonlinear
Below: fully implicit, fully coupled, but linear

