

Numerical simulation of flow induced vibrations of a vocal fold model with consideration of different boundary conditions

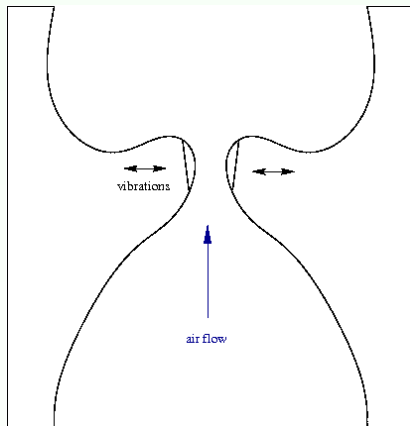
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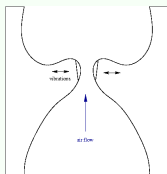
Vocal folds, vocal tract geometry:



vibrations of vocal folds - **simple tone with frequency f**

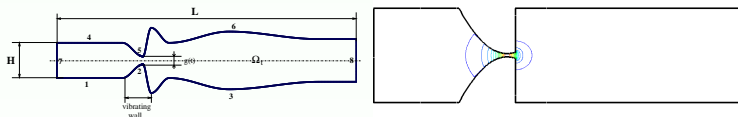
Research focuses on **development of a simple mathematical model** which allows to model basic qualitative and quantitative properties known from available data (lungs pressure, frequency of vibrations, flow volumes)

- Flanagan, Landgraf, one mass model (1968)
- Ishizaka, Elenagen, two mass model (rectangular, 1972)
- Pelorson, Hirschberg, modified two mass model (rounded, 1994)
- J. Horacek, P. Sidlof, J. Svec - further extension
- Our goal: to develop FE based methodology applicable for complicated geometries, interacting with (elastic) vocal folds vibrations induced by aerodynamics forces.
- A suitable benchmark for such a situation is missing, **compare simplified model with our FE results.**



- **air flow model in vocal tract:** low speeds (incompressible)
- **structure model:** vibrating, almost closing
- **interface conditions**

Here: *simplified coupled model for 2d flows around glottal part described by “three-mass” model*



Geometry

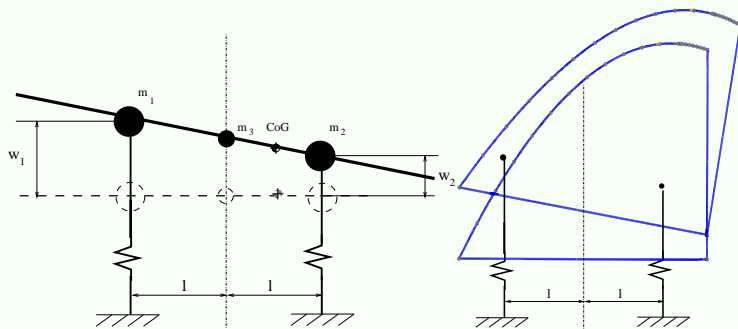
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A simplified model of glottal region of human vocal tract

Structure model

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Elastic structure motion - two mass model



$$\mathbb{M} \begin{pmatrix} \ddot{w}_1 \\ \ddot{w}_2 \end{pmatrix} + \mathbb{B} \begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} + \mathbb{K} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = 0$$

$$\mathbb{M} = \begin{pmatrix} m_1 + \frac{m_3}{4} & \frac{m_3}{4} \\ \frac{m_3}{4} & m_2 + \frac{m_3}{4} \end{pmatrix}, \quad \mathbb{K} = \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$$

- Navier-Stokes equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = 0$$
$$\nabla \cdot \mathbf{v} = 0 \quad \text{in } \Omega_t$$

- Mapping $\mathcal{A}_t : \Omega_0 \mapsto \Omega_t$
- Domain velocity (grid velocity)

$$\mathbf{w}_g(x, t) = \frac{\partial \mathcal{A}_t(\xi)}{\partial t} = \frac{\partial u}{\partial t}(\xi, t)$$

- ALE derivative - time derivative on ALE trajectory

$$\frac{D^{\mathcal{A}} f}{Dt} = \frac{\partial f}{\partial t} + (\mathbf{w}_g \cdot \nabla) f$$

- Navier-Stokes equations in ALE form

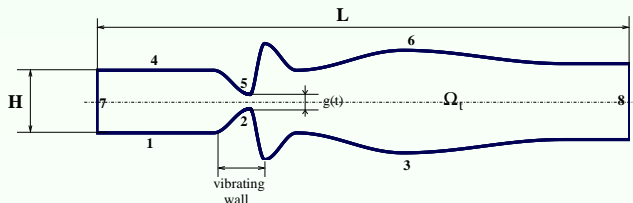
$$\frac{D^{\mathcal{A}} \mathbf{v}}{Dt} + ((\mathbf{v} - \mathbf{w}_g) \cdot \nabla) \mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = 0$$
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- Wall ($\mathbf{v} = 0$)
- Moving boundary ($\mathbf{v} = \mathbf{w}_g$)
- **Outlet boundary condition - modified** do-nothing

$$-\nu \frac{\partial \mathbf{v}}{\partial \mathbf{n}} + (p - p_{out})\mathbf{n} + \frac{1}{2}(\mathbf{v} \cdot \mathbf{n})^{-}\mathbf{v} = 0$$

- **Inlet boundary condition** Dirichlet ($\mathbf{v} = \dots$) or modified do-nothing (pressure)

$$-\nu \frac{\partial \mathbf{v}}{\partial \mathbf{n}} + (p - p_{in})\mathbf{n} + \frac{1}{2}(\mathbf{v} \cdot \mathbf{n})^{-}\mathbf{v} = 0$$

- time discretization - BDF2
- ALE mapping - solve BVP (for displacement)
- based on FEM, i.e. weak formulation
- space discretization - triangular FE (**Taylor-Hood** or P1/P1)
- Galerkin Least Squares, div-div stabilizations (ALE!)
- Solution of nonlinear problem: Oseen linearizations
- Solution of linear problems: direct or multilevel
- coupled solution: strong coupling (no relaxation needed)



M. Feistauer, J. Horáček, and P. Sváček.

Numerical simulation of flow induced airfoil vibrations with large amplitudes.

Journal of Fluids and Structure, 23(3):391–411, 2007.



P. Sváček and J. Horáček.

On Application of Stabilized Higher Order Finite Element Method on Unsteady Incompressible Flow Problems.

In Num Math and Advanced Applications. , pages 897–905, Berlin, 2006. Springer.

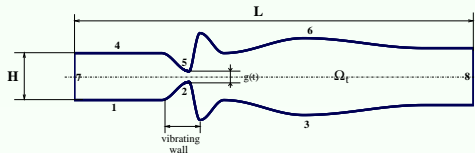
Numerical methods is realized by

- in-house software (C language)
- GNU C, Intel C compilers
- different platforms (Windows, Linux, Digital Unix, ...)
- use BLAS/LAPACK library
- use UMFPACK/MUMPS/PETSC library

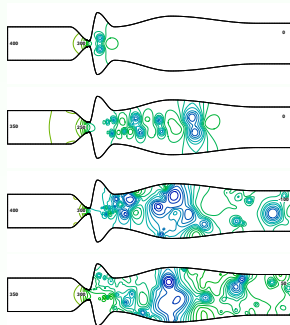
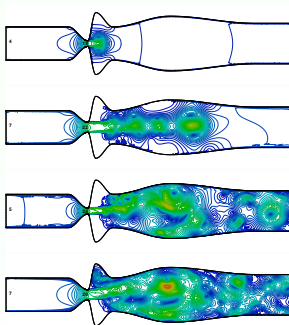
Numerical results

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Typical flow patterns in vocal tract (no structure)



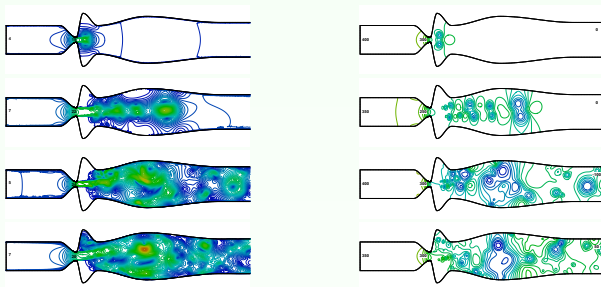
$$\Delta p = 400 \text{ Pa}, \quad Re \approx 10000 \quad g(t) \approx 4 \text{ mm}, \quad f = 100 \text{ Hz}.$$



Numerical results

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Typical flow patterns in vocal tract (no structure)

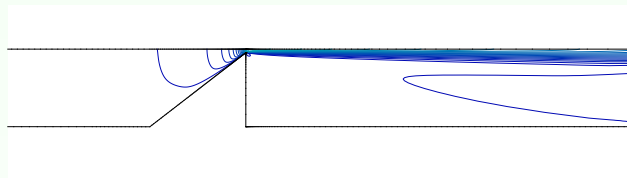


- complex flow patterns (due $Re \approx 10^4$), unsymmetric solution
- closing gap (thus: inlet velocity - non-realistic values)
- numerical results are hard to verify
- [Movie 1\(vel\)](#) [Movie 2\(vel\)](#)

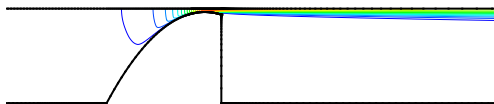
Numerical results

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Numerical approximation of an aeroelastic problem



“Female vocal folds geometry”



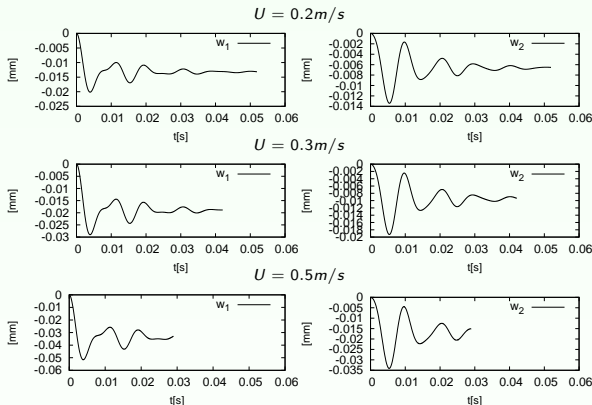
“Male vocal folds geometry”

Compare low-fidelity model, (Horacek, Svec, 2002)

Numerical results

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Female geometry, pressure drop

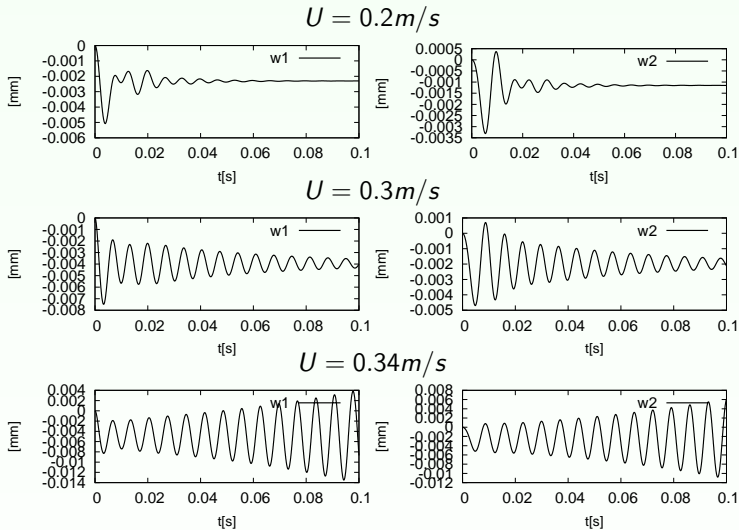


- pressure drop problem, U - resulting inlet velocity,
- ... with further increase of $\Delta p \rightarrow$ ael instability ...
- ... but for non-realistic values ($\Delta p = 10kPa$, $U_{max} \approx 180m/s$)
- whereas simpl. model: instability for $0.4 - 1.2m/s$ (gap)

Numerical results

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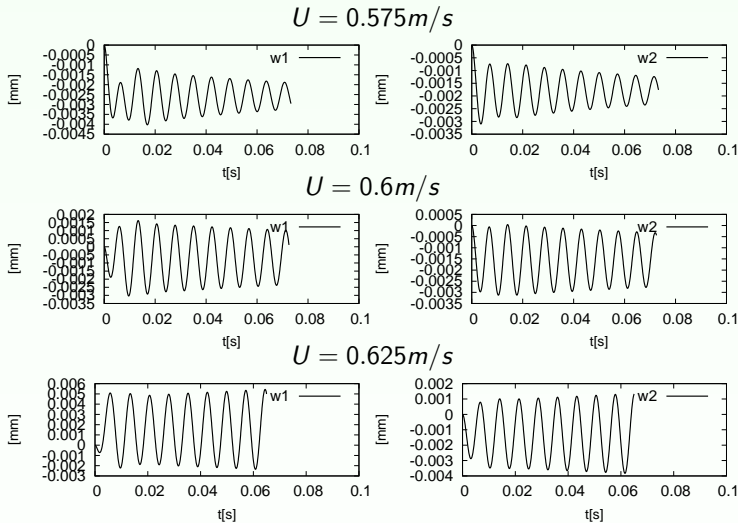
Female geometry, inlet velocity



Numerical results (“male” geometry)

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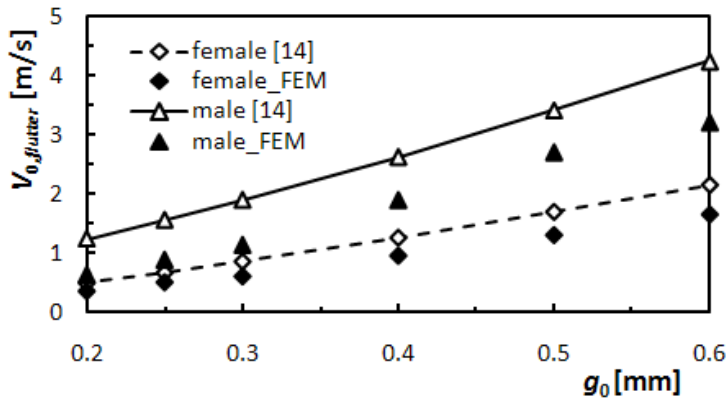
Male geometry, inlet velocity



Numerical results

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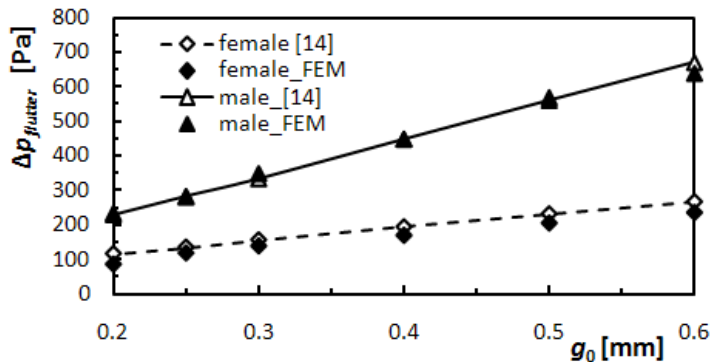
Quantitative comparison: flutter velocity



Numerical results

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Quantitative comparison: pressure



- Aeroelastic simulations of flow in the vibrating glottal part described by two mass model was presented.
- Numerical results obtained by **GLS stabilized FEM** were presented.
- The qualitative and quantitative results of the high-fidelity presented model were compared with low fidelity model.
- **The influence of the inlet/outlet boundary conditions was discussed.**
- The inlet velocity formulation agrees better with the low fidelity model, but the pressure drop formulation still have advantages - a combination of both b.c. can be used.