Radiative-Shock Solutions from Grey S_n-transport with Temperature- and Density-dependent Cross Sections

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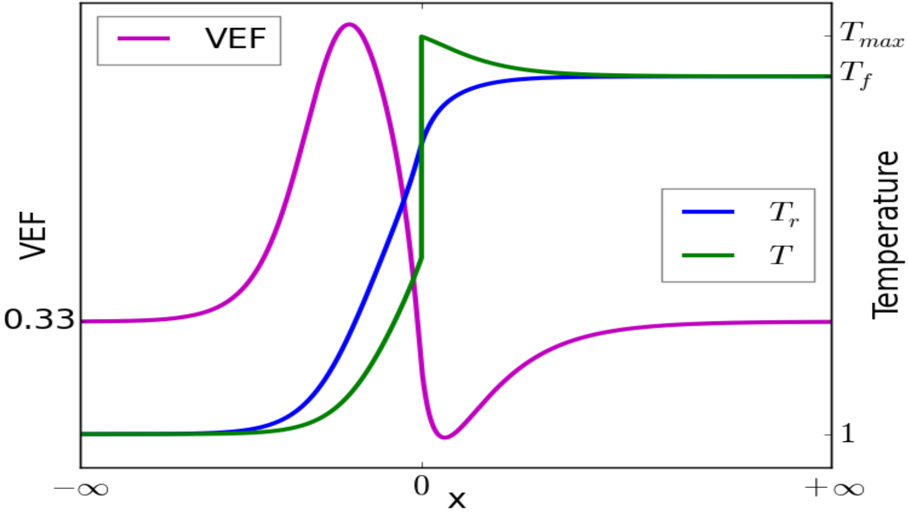
Overview

A generic shock solution



Assumptions

- The local material is sufficiently hot for radiation to affect the hydrodynamics $> 10^6[K] \sim 100[eV]$.
- Single material temperature.
- S_n radiation model.
- Grey, temperature- and density-dependent opacities and an ideal-gas γ-law EOS.
- An infinitely long shock-tube (thick-thick shocks).
- Material is non-relativistic.

What is a semi-analytic shock solution

- Relevant PDEs reduced to system of ODEs and integrated using a standard integrator with error control.
- Provides radiation hydrodynamic benchmark solutions assuming certain physics models.
- Improve our theoretical understanding:
  - Equilibrium Diffusion - Radiative shocks can be continuous for small and large values of the Mach number, $\mathcal{M}_0$.
  - Nonequilibrium Diffusion - A Zel'dovich spike may exist independently of the embedded hydrodynamic shock.
  - Radiative transfer - Anti-diffusive shocks exist for certain ranges of $\mathcal{M}_0$, which diffusion theory (i.e., $F_r \sim -\nabla E_r$) fails to model.
  - Bremstrahlung emission - Transmissive radiation-pressure wave, analogous to the Marshak solution, at moderate $\mathcal{M}_0$.

Previous approximate and semi-analytic solutions

- Sen and Guess (1957) - recommended S_n via Chandrasekhar
- Heaslet and Brown (1963) - $\mathcal{M}_0$ and P_0, weak to strong
- Ensman & Burrows (1994) - collected RT/RH test problems because solutions already forgotten
- Drake (2007) - ‘adaptation zone’, ‘transmissive’ ($f > 1/3$) and ‘diffusive’ ($f \approx 1/3$) precursor regions
- Lowrie and Rauenzahn (2007) - semi-analytic equil. (1-T) diff.
- Lowrie and Edwards (2008) - semi-analytic nonequil. (2-T) diff.
- McClarren and Drake (2010) - analytic anti-diff. ($F_r \not\propto -\nabla E_r$)

RH equations and the EOS

The 1-D nondim. steady-state radiation-coupled Euler equations are

$$\partial_x \left(\rho u\right) = 0 \,,$$

$$\partial_x \left(\rho u^2 + p_m\right) = -P_0 S_{rp} \,,$$

$$\partial_x \left[u \left(\frac{1}{2}\rho u^2 + \rho e + p_m\right)\right] = -P_0 C_0 S_{re} \,,$$

with an ideal-gas EOS, $p_m = (\gamma - 1)\,\rho e \quad \& \quad e = \frac{T}{\gamma(\gamma-1)} \quad \& \quad \gamma = \frac{5}{3}$, and the radiation-transport equation, correct through $\mathcal{O}\left(\beta\right)$ with $\mathcal{O}\left(\beta^2\right)$ equilibrium-source corrections, is

$$\mu \partial_x I = -\sigma_t I + \frac{\sigma_s}{4\pi} E_r + \frac{\sigma_a}{4\pi} T^4 - 2\frac{\sigma_s}{4\pi}\beta F_r + \beta\mu\left(\sigma_t I + \frac{3\sigma_s}{4\pi} E_r + \frac{3\sigma_a}{4\pi} T^4\right)$$
$$+ \frac{1}{4\pi}\beta^2 \left(\left(\sigma_s - \sigma_a\right)\left(E_r + P_r\right) + \sigma_a \left(T^4 - E_r\right)\right) \equiv Q\left(\mu\right) \,.$$

The Radiation Moment Equations

- The radiation energy and momentum source equations are obtained by taking the zero'th and first angle-integrated angular moments of the grey transport equation:

$$S_{re} = \int_{4\pi} Q(\mu)\, d\mu = \frac{\partial F_r}{\partial x},$$
$$S_{rp} = \int_{4\pi} \mu Q(\mu)\, d\mu = \frac{\partial P_r}{\partial x},$$
$$\vdots$$
$$\frac{\partial P_r}{\partial x} = \frac{\partial (f E_r)}{\partial x} \not\approx f \frac{\partial E_r}{\partial x} \sim \frac{\Delta E_r}{\Delta x_{BIGGER}} \otimes$$

- These moment equations are closed by saying $P_r = f\, E_r$, where $f(x) \in (0, 1]$ is called the variable Eddington factor (VEF).
- We solve the transport equation to determine a new f.
- This suggests a straightforward global iterative solution procedure.

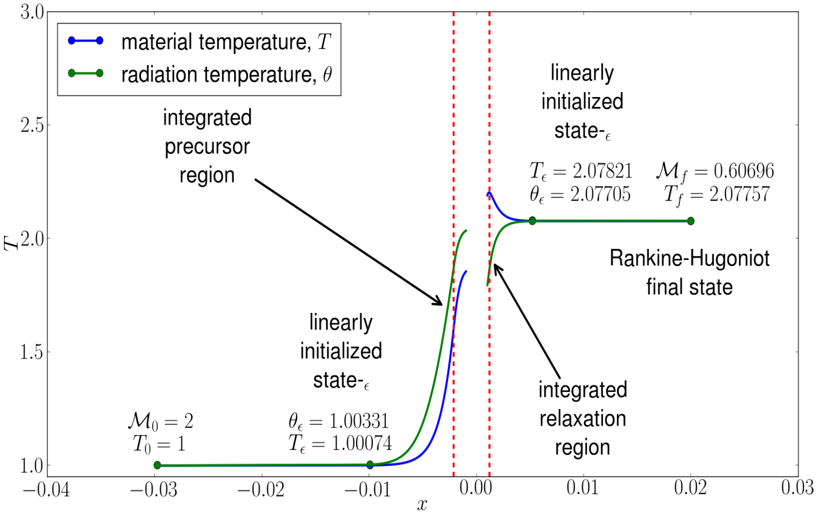
Global Solution Algorithm

- Overall solution process is iterative in 2 steps:
  1. RAD-HYDRO SOLVE
     - Begin with solution algorithm of Lowrie and Edwards (2008). (described on the next two slides)
     - Assume $f = 1/3$, or use updated VEF from step 2. below.
     - Solve "reduced" RH equations (Euler plus rad energy and momentum equations using the EF/VEF). This gives profiles for T, ρ, p, ..., and E_r, F_r, and P_r.
  2. Sn SOLVE
     - Use variables from rad-hydro solve to construct right-hand side of the transport equation: $\mu \partial_x I + \sigma_t (1 - \beta\mu) I = q$.
     - Perform sweep (invert left-hand side S_n operator using ODE solver with error control). This gives profiles for E_r, F_r, P_r and $f = P_r/E_r$.
- Repeat 1. and 2. until the two versions of E_r, F_r, and P_r agree.

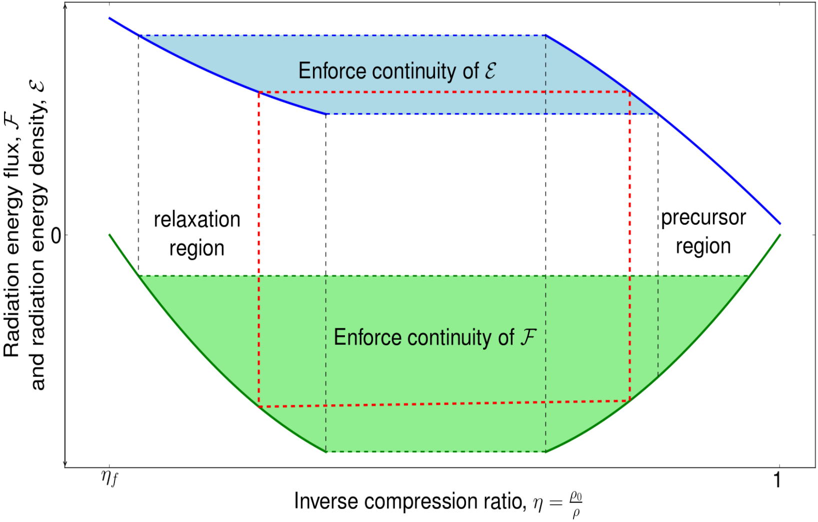
Reduced-System Solution Algorithm

- Reduce the system of equations to two ODEs.
- Define upstream conditions at $x = -\infty$.
- Derive downstream final conditions at $x = +\infty$ using continuity of flux (Rankine-Hugoniot conditions).
- Linearize away from upstream and downstream equilibrium states.
- Integrate away from upstream state toward downstream state, and separately, integrate away from downstream state toward upstream state.
- Connect these two solutions to obtain the shock profile by enforcing continuity of the lab-frame radiation flux.

Shock profile solution procedure



Shock profile solution procedure



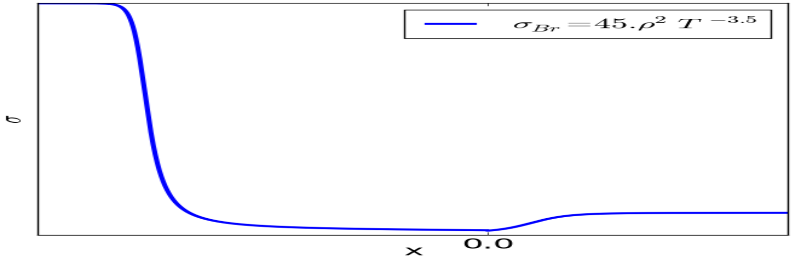
Bremstrahlung absorption

Equation 5.24, Zel'dovich & Raizer:

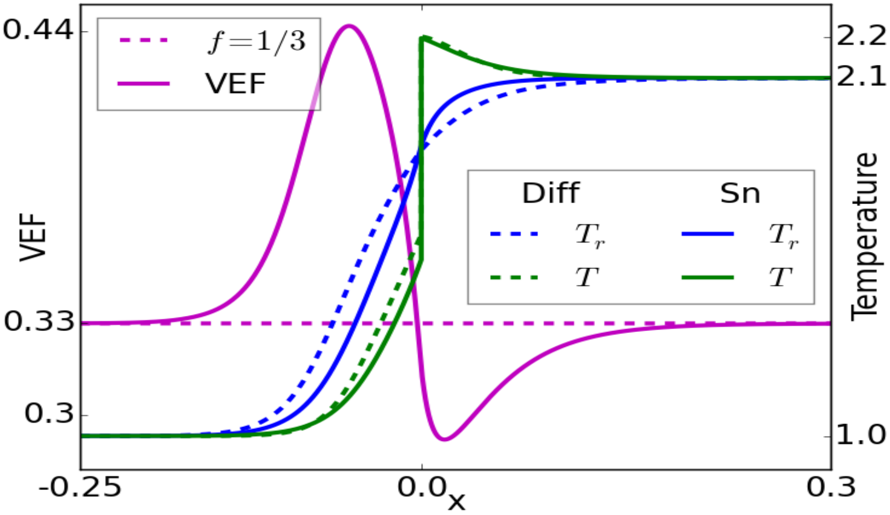
$$\sigma_{Br} \approx 45 \frac{\rho^2}{T^{7/2}} \left[cm^{-1} \right] , \quad \rho = \left[\frac{g}{cc} \right] , \quad T = [eV] \ .$$

Frequency-dependent derivation given in Landau & Lifshitz, vol 2.

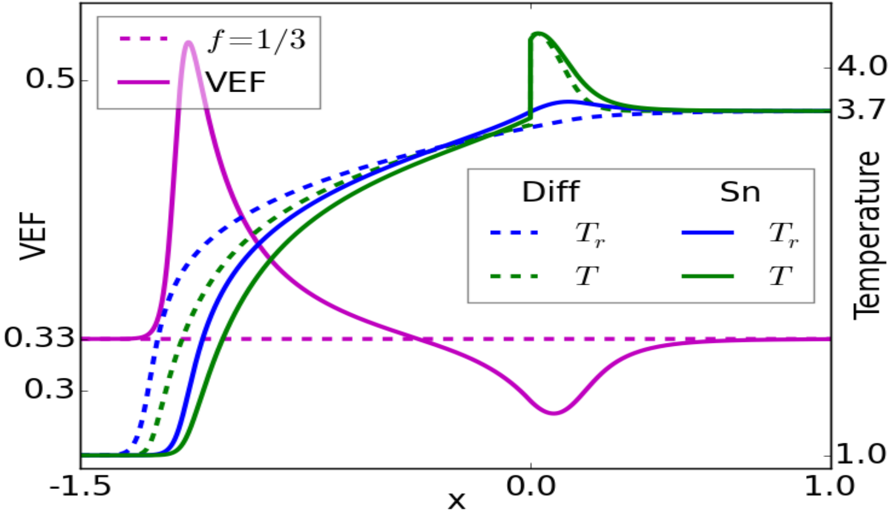
For all Mach numbers $\rho_f \sim 1 - 7$ and $T_f \sim M^n$, so σ_{Br} dominated by T.



$\mathcal{M}_0 = 2$ Comparison to nonequilibrium diffusion



$\mathcal{M}_0 = 3$ The VEF is steepening



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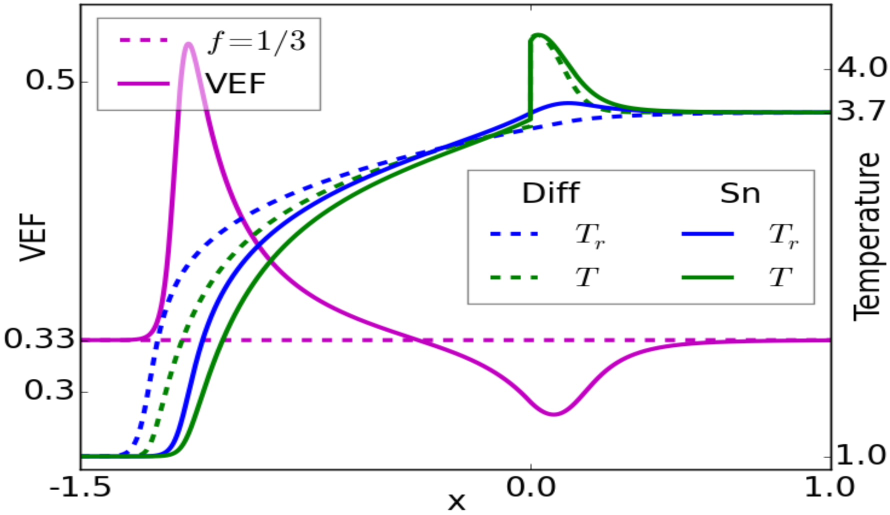
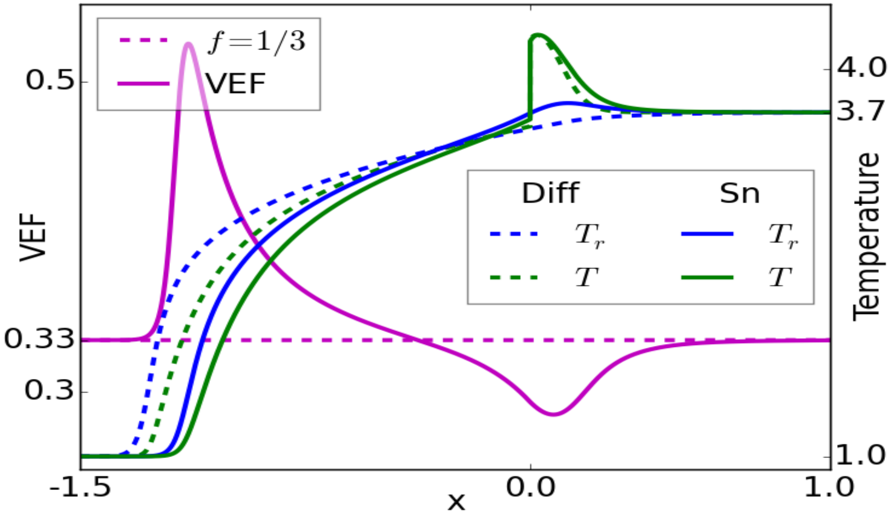
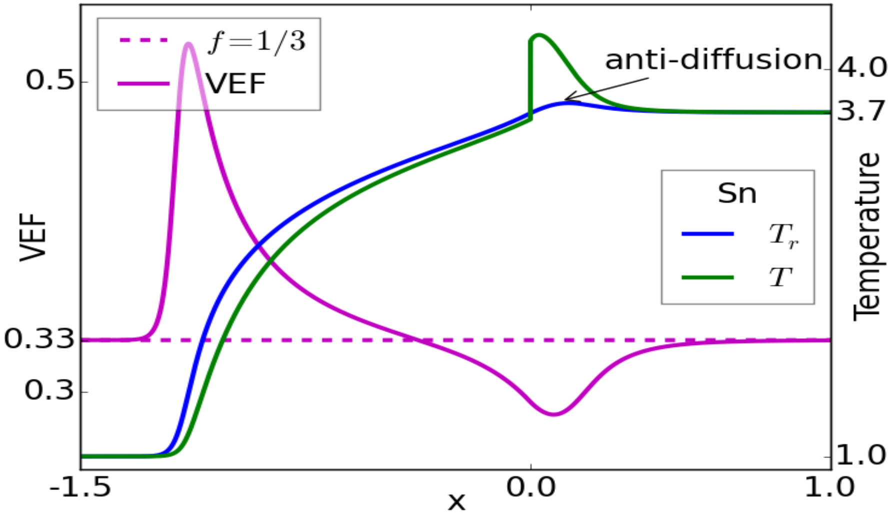


$\mathcal{M}_0 = 3$ Adaptation zone



$\mathcal{M}_0 = 3$ Anti-diffusion: $F_r \not\approx -\nabla E_r = -\nabla Tr^4$



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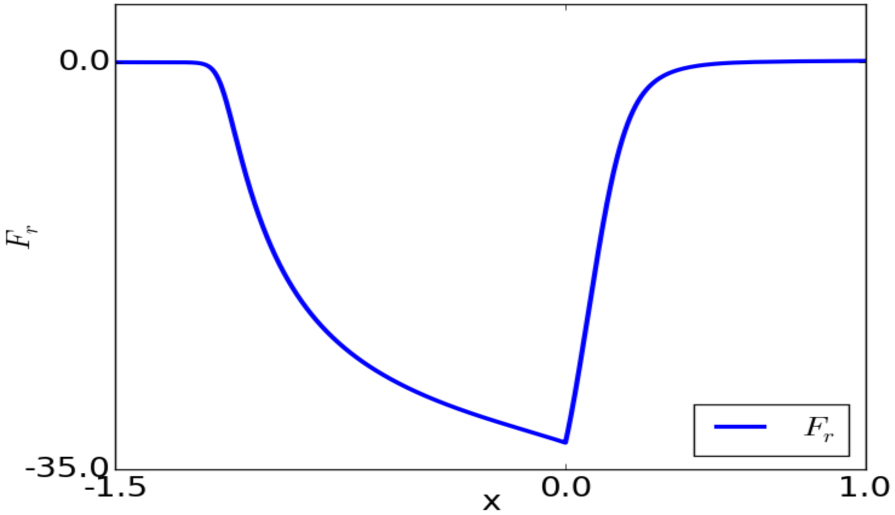
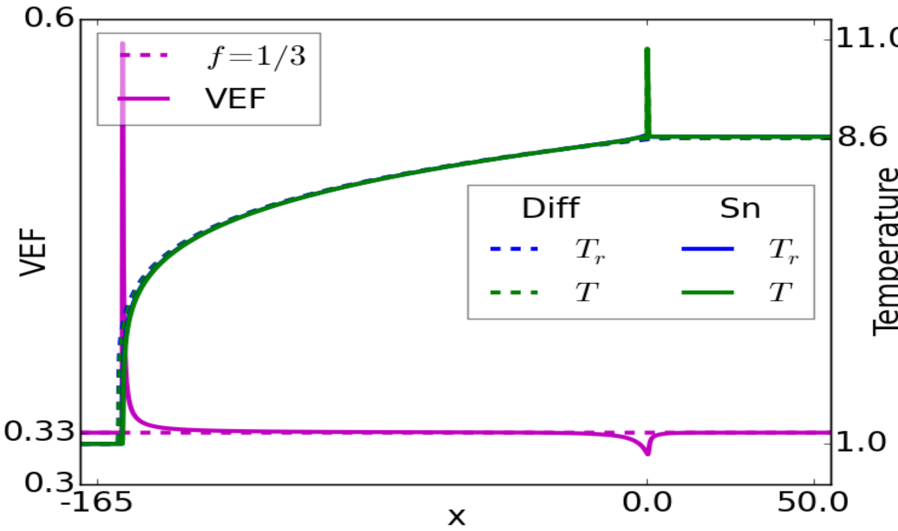
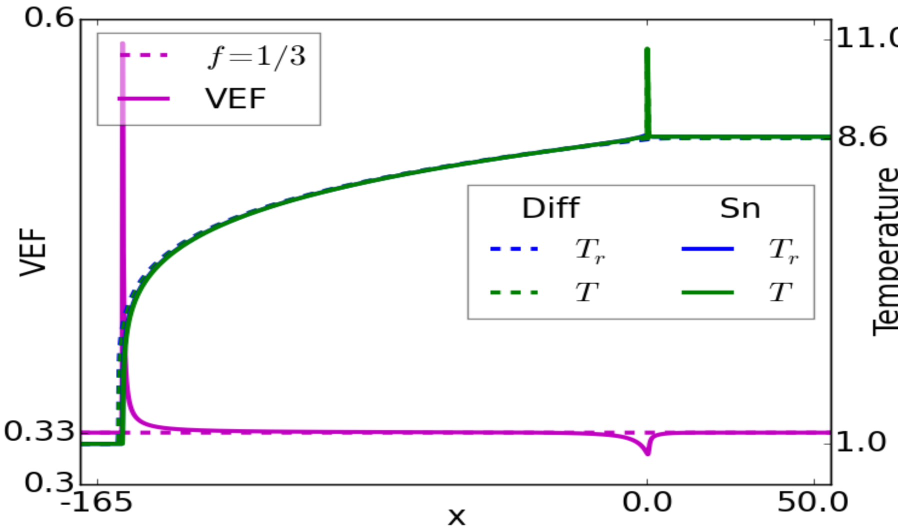


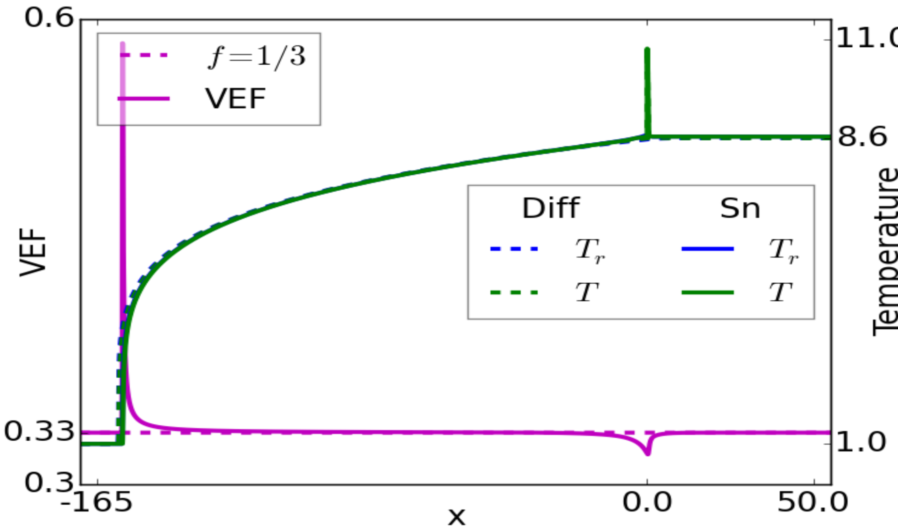
$\mathcal{M}_0 = 5$ The VEF is very steep



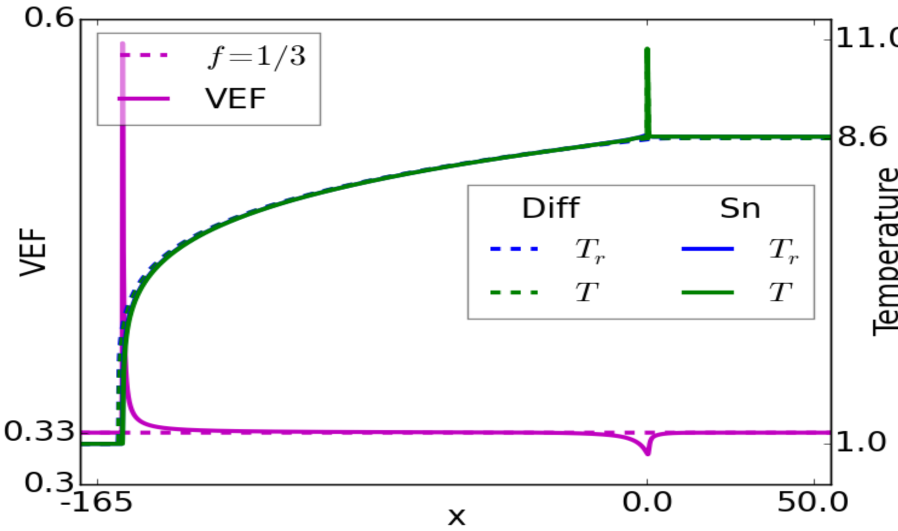
$\mathcal{M}_0 = 5$ Adaptation zone



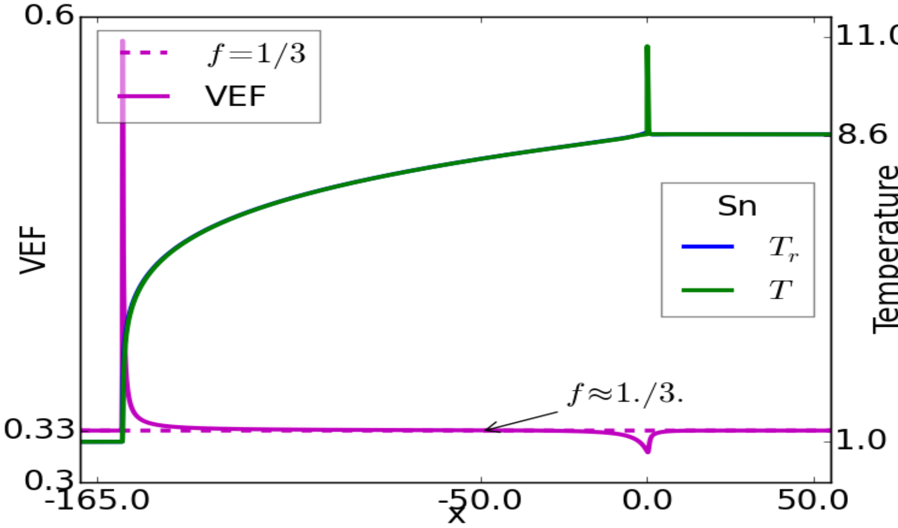
$\mathcal{M}_0 = 5$ Comparison to nonequilibrium diffusion



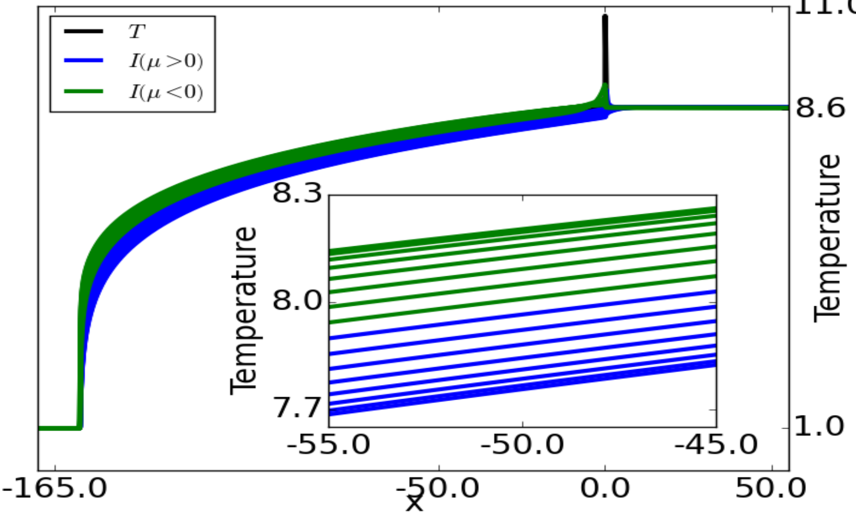
Diffusion may be good enough, if $\mathcal{M}_0$ is large enough



Equilibrium & diffusion imply isotropy. How isotropic?

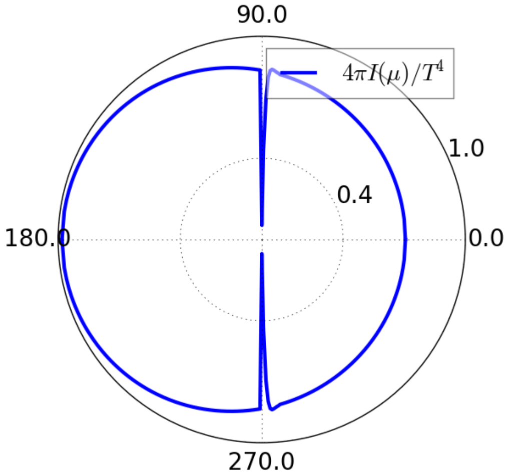


A quantitative measure of isotropy



All S_n rays should live on the analytic polar plot

$$I_\mu(x) = I_{eq} e^+_{eq} e^- + e^- \int_{x_{eq}}^{x} \frac{q'_\mu(E'_r, F'_r, P'_r)}{\mu} e^{+\prime} dx' \,, \qquad e^\pm \equiv e^{\pm \frac{\sigma_t}{\mu}(1-\beta\mu)x}$$



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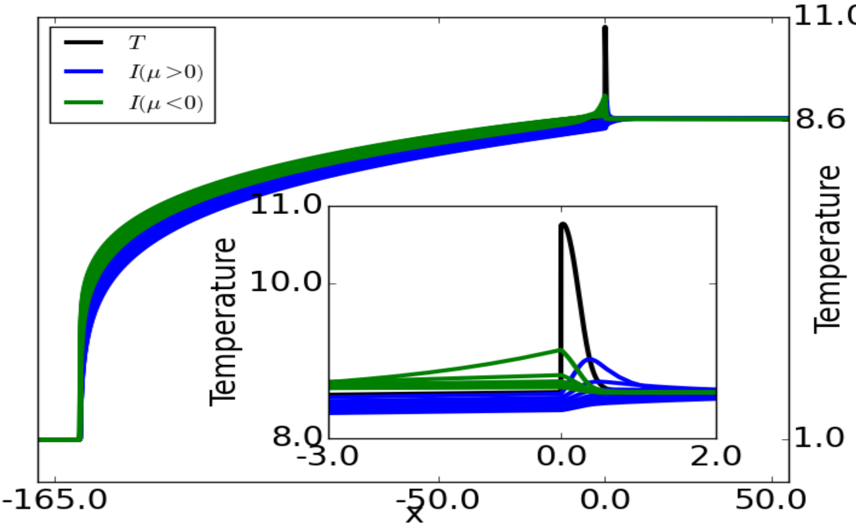
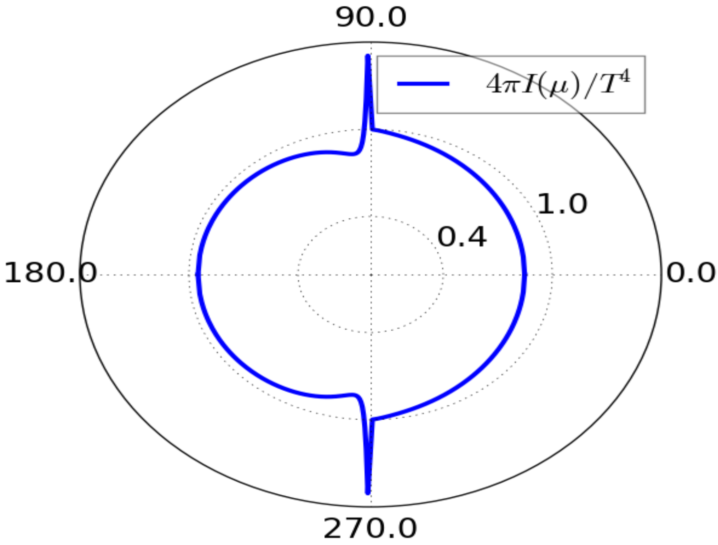
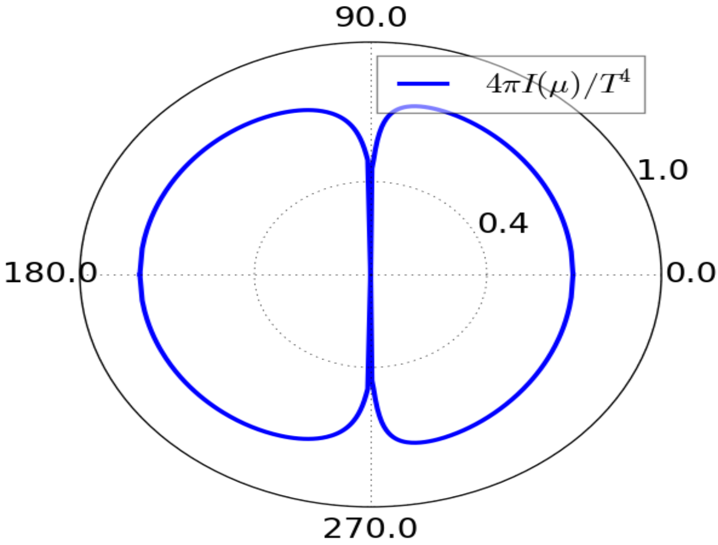


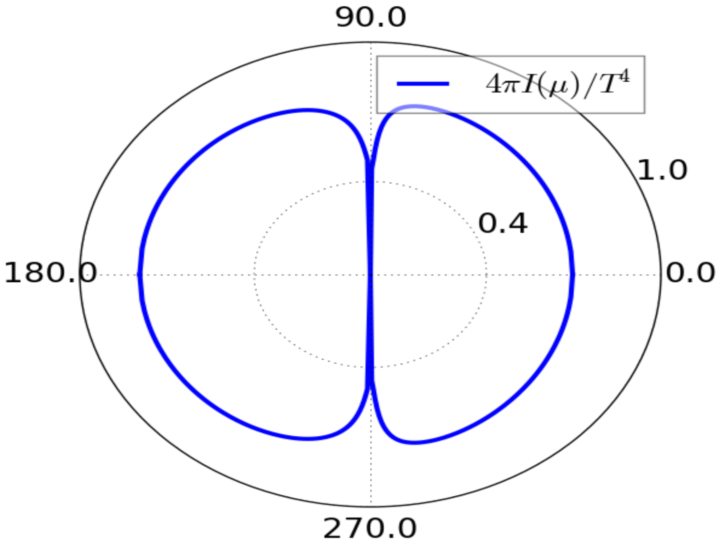
At $x = 0$, the angular distribution peaks about $\mu = 0$



At $x = 0^+$, the angular distribution avoids $\mu = 0$



Any S_n choice should match this result



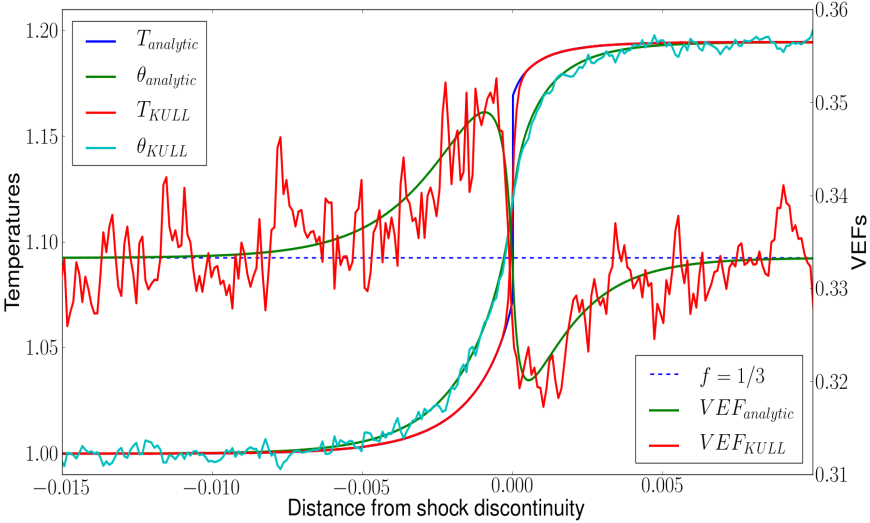
Conclusions

- Presented grey, temperature- and density-dependent opacities, for semi-analytic S_n-transport radiative-shock solutions. These solutions are a useful code-verification tool of RH codes that solve the radiation transport equation.
- Diffusion (incorrectly) pushes the shockfront further into the gas.
- Adaptation zone exists on the precursor side, adjacent to the hydrodynamic shock.
- Anti-diffusion exists for shocks with Bremstrahlung emission.
- VEF becomes spike at shockfront as $\mathcal{M}_0$ increases.
- Angular distributions are peaked about $\mu = 0$ at the hydrodynamic shock ($x = 0$), and appear to avoid $\mu = 0$ away from the hydrodynamic shock.
- Transmissive & diffusive regions exist.

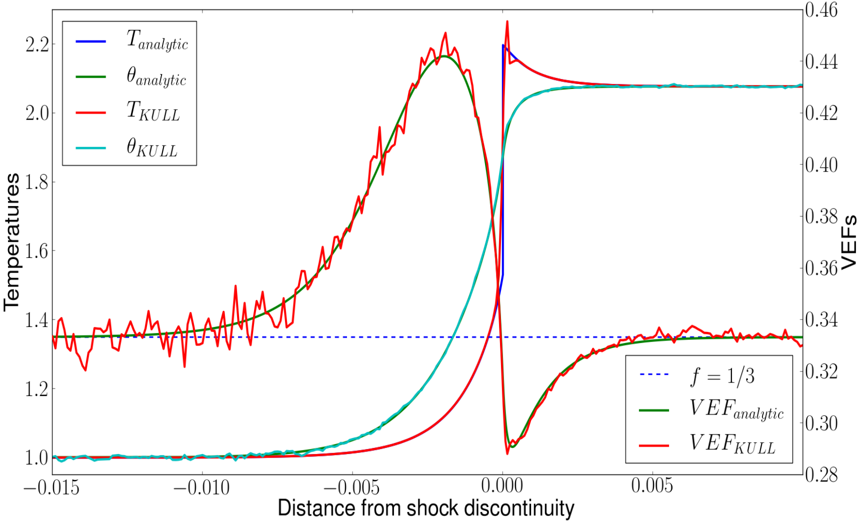
Future Work

- Incorporate fully-relativistic radiation transport as well as fully-relativistic material motion.
- Incorporate frequency-dependent diffusion and frequency-dependent transport.
- Zel'dovich & Raizer claim Bremstrahlung radiation is dominated by decelerating high-velocity electrons, from the tail of the Maxwell distribution, generating high-frequency radiation.
- Incorporate separate elecron and ion temperatures.
- Investigate validity of various material-motion models for radiation.

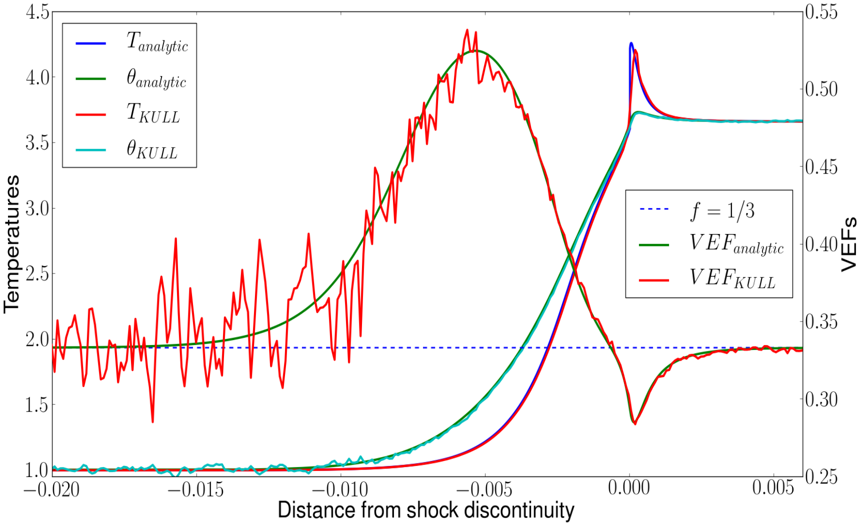
$\mathcal{M}_0 = 1.2$ Comparison with Fully Relativistic IMC



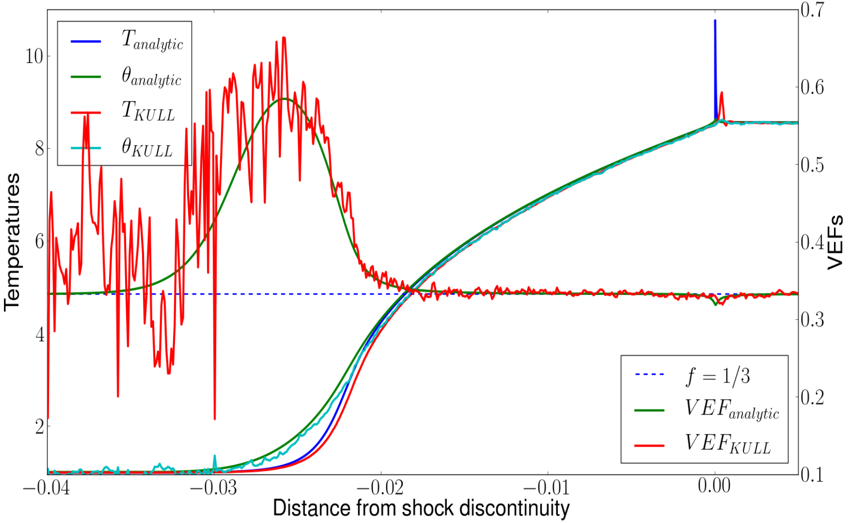
$\mathcal{M}_0 = 2$ Comparison with Fully Relativistic IMC



$\mathcal{M}_0 = 3$ Comparison with Fully Relativistic IMC



$\mathcal{M}_0 = 5$ Comparison with Fully Relativistic IMC



$\mathcal{M}_0 = 1.2$ Comparison with Fully Rel. Astro-code

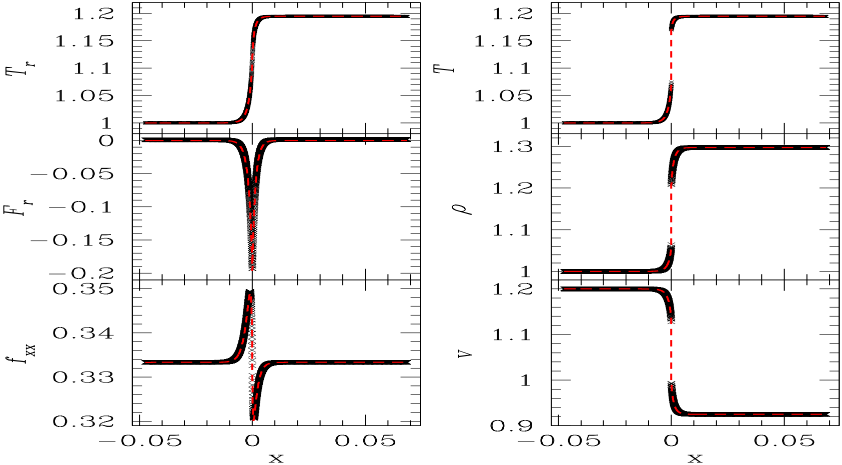


Figure 11. Structure of a radiation-modified shock for Mac number $\mathcal{M} = 1.2$. The dashed red lines are the semi-analytic solution by solving the time-independent radiation hydrodynam equations while the black dots are the numerical results when th flow reaches steady state.

$\mathcal{M}_0 = 2$ Comparison with Fully Rel. Astro-code

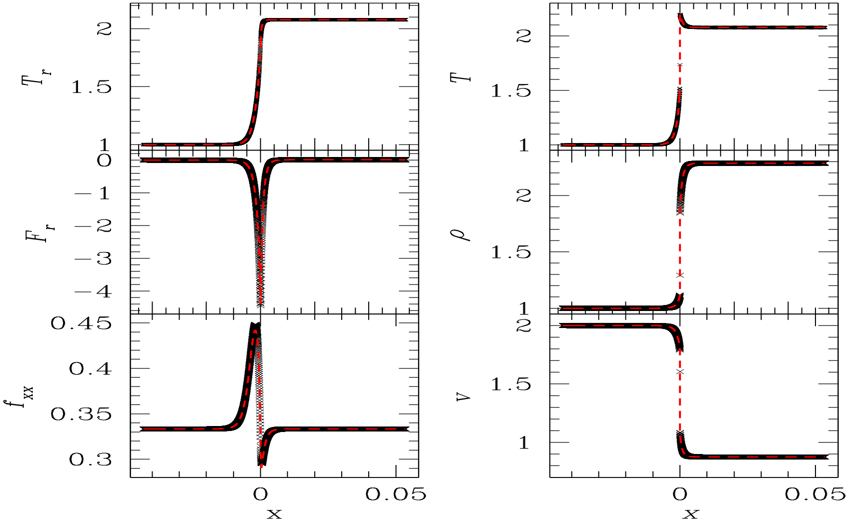


Figure 12. The same as Figure 11 but for Mach number $\mathcal{M} =$ 2

$\mathcal{M}_0 = 3$ Comparison with Fully Rel. Astro-code

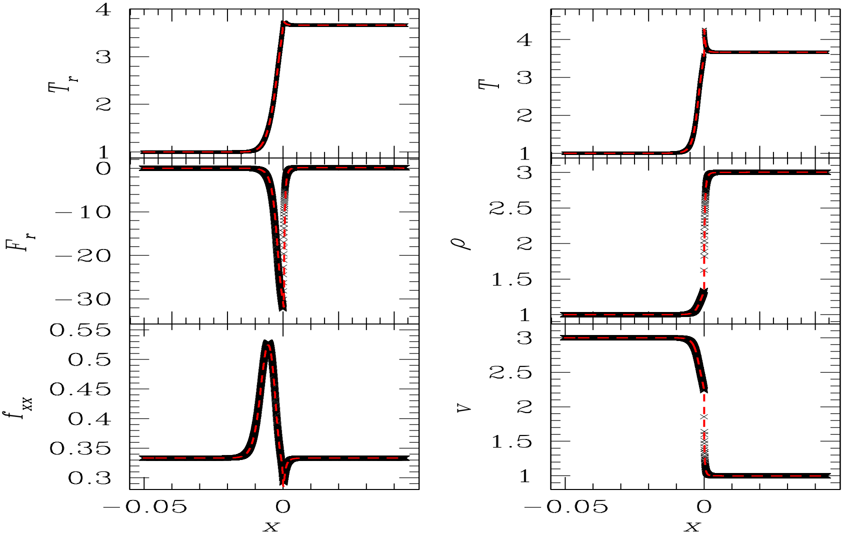


Figure 13. The same as Figure 11 but for Mach number $\mathcal{M} =$