A Multimaterial Direct ALE cell-centered solver for the simulation of complex flows.

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Physical Model - Monomaterial description

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \mathrm{dV}\right)+\int_{A(t)} \overrightarrow{w}\cdot\overrightarrow{\nu}\ \mathrm{dS}=0$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho\ \mathrm{dV}\right)+\int_{A(t)} \rho\overrightarrow{\mu}\cdot\overrightarrow{\nu}\ \mathrm{dS}=0\,,$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho\overrightarrow{v}\ \mathrm{dV}\right)+\int_{A(t)} \rho\overrightarrow{v}\ \overrightarrow{\mu}\cdot\overrightarrow{\nu}\ \mathrm{dS}+\int_{A(t)} p\overrightarrow{\nu}\ \mathrm{dS}=0\,,$$

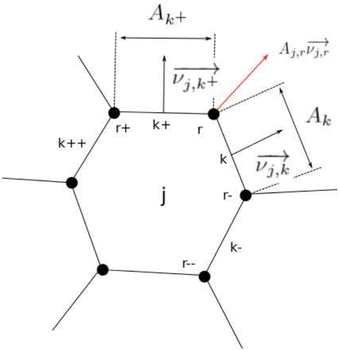
$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho E\ \mathrm{dV}\right)+\int_{A(t)} \rho(t)H\,\overrightarrow{\mu}\cdot\overrightarrow{\nu}\ \mathrm{dS}=0\,,$$

with $H=E+\dfrac{p}{\rho}$ (enthalpy), $e=E-\frac{1}{2}\overrightarrow{v}^2$ (internal energy), $\overrightarrow{\mu}=\overrightarrow{v}-\overrightarrow{w}$ (Relative velocity) and :

$$p=f(\rho,e)\ \ (\mathrm{EOS})$$

Notations

Specific indices $\{j, r, k\}$ will denote cells, nodes and faces.



- A_k : in 2D, length of the edge 'k'
- $\overrightarrow{\nu_{j,k}}$: outward-pointing normal at the edge 'k'

- $\mathcal{N}$: nodes of the mesh
- $\mathcal{C}$: cells of the mesh
- $\mathcal{E}$: edges of the mesh
- $\mathcal{E}(j)$: set of edges bounding the cell 'j'.
- $\mathcal{N}(j)$: set of nodes defining the cell 'j'.
- $\mathcal{C}(r)$: set of cells sharing the node 'r'.

2D Numerical Scheme - Monomaterial description

We consider a robust Lagrangian scheme (here GLACE or EUCCLHYD with a face-based formulation of the numerical fluxes) in which we add upwinded advection fluxes :

$$\frac{\mathrm{d}}{\mathrm{dt}}(V_j) + \sum_{k \in \mathcal{E}(j)} A_k \overrightarrow{w_k} \cdot \overrightarrow{\nu_{j,k}} = 0 \,, \mathrm{GCL}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}(\rho_j V_j) + \sum_{k \in \mathcal{E}(j)} A_k \rho_k^{\mathrm{up}} \, \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} = 0 \,, \mathrm{mass}$$

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}(\rho_j V_j \overrightarrow{v_j}) + \sum_{k \in \mathcal{E}(j)} A_k \rho_k^{\mathrm{up}} \overrightarrow{v_k^{\mathrm{up}}} \, \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} \\ + \sum_{k \in \mathcal{E}(j)} A_k p_{j,k} \cdot \overrightarrow{\nu_{j,k}} = 0 \end{aligned}\right. \,, \mathrm{momentum}$$

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}(\rho_j V_j E_j) + \sum_{k \in \mathcal{E}(j)} A_k \rho_k^{\mathrm{up}} E_k^{\mathrm{up}} \, \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} \\ + \sum_{k \in \mathcal{E}(j)} A_k p_{j,k} \, \overrightarrow{v_k} \cdot \overrightarrow{\nu_{j,k}} = 0 \end{aligned}\right. \,, \mathrm{total\ energy}$$

Remark : the Lagrangian scheme must ensure the compatibility between the motion of the nodes and the GCL.

Lagrangian fluxes

Face - Node relations :

$$A_{j,r}\overrightarrow{\nu_{j,r}} = \frac{1}{2}\left(A_k\,\overrightarrow{\nu_{j,k}} + A_{k+}\,\overrightarrow{\nu_{j,k+}}\right)\left(\sum_{j\in\mathcal{C}(r)} A_{j,r}\overrightarrow{\nu_{j,r}} = 0\right) \tag{1}$$

$$\overrightarrow{v_k} = \frac{1}{2}\left(\overrightarrow{v_r} + \overrightarrow{v_{r+}}\right)\ , \tag{2}$$

$$p_{j,k} = \frac{1}{2}\left(p_{j,r} + p_{j,r+}\right)\ , \tag{3}$$

$$p_{j,k}\overrightarrow{v_k} = \frac{1}{2}\left(p_{j,r}\,\overrightarrow{v_r} + p_{j,r+}\,\overrightarrow{v_{r+}}\right)\ . \tag{4}$$

Nodal pressures are computed from the nodal velocities :

$$p_{j,r}\overrightarrow{\nu_{j,r}} = p_j\overrightarrow{\nu_{j,r}} - \bar{\bar{\alpha}}_r\left(\overrightarrow{v_r} - \overrightarrow{v_j}\right)\ ,$$

where the acoustic tensor $\bar{\bar{\alpha}}_r \in \mathbb{R}^2 \times \mathbb{R}^2$ is a symmetric positive-definite matrix depending on the chosen Lagrangian scheme :

GLACE	EUCCLHYD
$\bar{\bar{\sigma}}_{j,r} = \rho_j\,c_j\,A_{j,r}\,\overrightarrow{\nu_{j,r}} \otimes \overrightarrow{\nu_{j,r}}$	$\bar{\bar{\sigma}}_{j,r} = \frac{\rho_j\,c_j}{A_{j,r}}\left(A_{j,k}\,\overrightarrow{\nu_{j,k}} \otimes \overrightarrow{\nu_{j,k}} + A_{j,k+}\,\overrightarrow{\nu_{j,k+}} \otimes \overrightarrow{\nu_{j,k+}}\right)$

Nodal Solver

Solving the system :

$$\sum_{j \in \mathcal{C}(r)} A_{j,r} p_{j,r} \overrightarrow{\nu_{j,r}} = 0 \tag{5}$$

for all internal nodes gives the nodal velocities :

$$\left(\sum_{j \in \mathcal{C}(r)} A_{j,r} \bar{\bar{\alpha}}_r \right) \overrightarrow{v_r} = \sum_{j \in \mathcal{C}(r)} A_{j,r} p_j \overrightarrow{\nu_{j,r}} + \sum_{j \in \mathcal{C}(r)} A_{j,r} \bar{\bar{\alpha}}_r \overrightarrow{v_j} \tag{6}$$

The boundary conditions are taken into account in the previous system by making some minor modifications in the previous system → just refer to the method used to solve the system above for each specific boundary condition (Neumann, imposed pressure, plane wall. . .)

Upwind method for the advective fluxes

The upwinded quantities are chosen regarding the sign of the face relative velocity $\overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}}$. We write for $\chi = \{\rho, \overrightarrow{v}, E\}$:

$$\chi_k^{\mathrm{up}} = \frac{1}{2}\left(\widetilde{\chi_j} + \widetilde{\chi_{j+1}}\right) + \mathrm{sign}\left(\overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}}\right)\left(\widetilde{\chi_j} - \widetilde{\chi_{j+1}}\right)$$

with $\mathrm{sign}(x) \in \{-1,\, 0,\, 1\}$ is the sign of the real number x.

1st order	2nd order
$\widetilde{\chi_j} = \chi_j$	$\widetilde{\chi_j} = \chi_j + L_j \left(\overrightarrow{\nabla}(\chi)_j\right)^t (\overrightarrow{x_k} - \overrightarrow{x_j})$ $\left[\overrightarrow{\nabla}(\chi)\right]_j = \underset{(\overrightarrow{\nabla}(\chi)) \in \mathbb{R}^2}{\arg\min} \sum_{l \in \mathcal{C}(j)} \left[\chi_l - \left(\chi_j + \overrightarrow{\nabla}(\chi) \cdot (\overrightarrow{x_l} - \overrightarrow{x_j})\right)\right]^2$ L_j : Venkatakrishnan limiter

2nd law of thermodynamics

Proposition

In any cell 'j', a semi-discrete formulation of the entropy balance is :

$$T_j \frac{\mathrm{d}\left(\rho_j V_j s_j\right)}{\mathrm{dt}} + T_j \sum_{k\in\mathcal{E}(j)} A_k \rho_k^{up} s_k^{up} \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} =$$

$$-\frac{1}{2} \sum_{k\in\mathcal{E}(j)} A_k \left(\overrightarrow{v_k^{up}} - \overrightarrow{v_j}\right)^2 \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} - \sum_{k\in\mathcal{E}(j)} A_k \left(p_{j,k} - p_j\right) \left(\overrightarrow{v_k} - \overrightarrow{v_j}\right) \cdot \overrightarrow{\nu_{j,k}} \,. \tag{7}$$

where T_j is the temperature in the cell 'j'. Providing that $\forall r \in \mathcal{N}(j)$, $\bar{\bar{\sigma}}_r$ is a symmetric positive-definite matrix, then our scheme produces entropy in the sense that :

$$\forall j \in \mathcal{C}(r), \ \ , T_j \frac{\mathrm{d}\left(\rho_j V_j s_j\right)}{\mathrm{dt}} + T_j \sum_{k\in\mathcal{E}(j)} A_k \rho_k^{up} s_k^{up} \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} \geq 0 \tag{8}$$

Stability - CFL condition

Our scheme is a first-order method in time and explicit in time. The quantities involved in the numerical fluxes are expressed at time t^n and the time derivatives are discretized as follows :

$$\frac{\mathrm{d}}{\mathrm{dt}}(\chi_j) = \frac{\chi_j^{n+1} - \chi_j^n}{\Delta t} \,. \tag{9}$$

We take the following CFL condition :

$$\max_{j \in \mathcal{C}} \left(\frac{\max\left(|v_x| + |w_x|\,, |v_y| + |w_y|\right) + c_j}{\Delta x_j} \right) \Delta t \leq \mathrm{CFL}\,, \tag{10}$$

with $\overrightarrow{v} = (v_x\,, v_y)^t$, $\overrightarrow{w} = (w_x,\, w_y)^t$, $CFL \in [0;1]$ and c_j the sound speed in the cell 'j', and the length Δx_j is arbitrarily computed as the ratio of the volume over the smallest face area among the surrounding faces bounding the cell 'j' :

$$\Delta x_j = \frac{V_j}{\min\limits_{k \in \mathcal{E}(j)} A_k} \,. \tag{11}$$

Computational method for the grid velocities

We use the L.E.L. method (Large Eddy Limitation) developed by Costes and Ghidaglia (talk on wednesday). The Hodge decomposition of the velocity field $\vec{v} : \Omega \to \mathbb{R}^2$ reads :

$$\vec{v} = (v_x\,, v_y)^T = \vec{\nabla}\, g + \vec{\nabla}^{\perp} f\,, \tag{12}$$

where

$$\vec{\nabla}^{\perp} f \equiv \left(-\frac{\partial f}{\partial y}, \frac{\partial f}{\partial x} \right)\,, \tag{13}$$

with f is computed from the Laplacian problem :

$$\begin{cases} -\Delta f & = -\vec{\nabla} \wedge \left(\vec{v}\right)\,, \text{ in } \Omega \\ f & = 0\,, \text{ on } \partial\Omega\,, \end{cases} \tag{14}$$

and $\vec{\nabla}\, g$ deduced from (12) : $\vec{\nabla}\, g = \vec{v} - \vec{\nabla}^{\perp} f\,.$

Remark : in 2D, $\vec{\nabla} \wedge (\ )$ is defined as :

$$\vec{\nabla} \wedge (\vec{v}) = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\,, \tag{15}$$

(2D) Computational method for the grid velocities

In pratice, the grid velocity is computed as follow : :

$$w(x) = \theta_g(x)\vec{\nabla}\, g(x) + \theta_f(x)\,\vec{\nabla}^{\perp} f(x)\,, \tag{16}$$

where the limiters are taken as :

$$\theta_g(x) = \varphi_g\left(|\vec{\nabla} g(x)|\right)$$

$$\theta_f(x) = \varphi_f\left(|\vec{\nabla}^{\perp} f(x)|\right)$$

and the function φ is chosen as :

$$\varphi(r) \equiv \min(1\,, \frac{\chi}{r})$$

with $\chi = \{\chi_g, \chi_r\}$ the limitation thresholds.

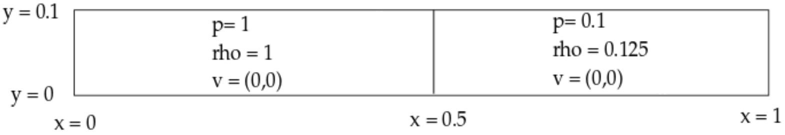
Limitation

The limitation thresholds values χ_g and χ_f uniformly limit the grid velocity within Ω.

χ_g	χ_f	θ_g	θ_f	$\overrightarrow{w}$	simulation
–	–	1	1	$\overrightarrow{w} = \overrightarrow{v}$	Lagrangian method
0	0	0	0	$\overrightarrow{w} = \overrightarrow{0}$	Eulerian method
any	0	$\varphi\left(\lvert\overrightarrow{\nabla} g\rvert\right)$	0	$\overrightarrow{w} = \theta_g \overrightarrow{\nabla} g$	limited compression / removed rotation
0	any	0	$\varphi\left(\lvert\overrightarrow{\nabla}^{\perp} f\rvert\right)$	$\overrightarrow{w} = \theta_f \left(\overrightarrow{\nabla}^{\perp} f\right)$	limited rotation / removed compression

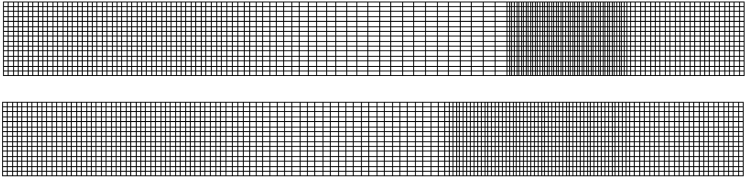
In the following results, we will always precise the values for χ_g and χ_f.

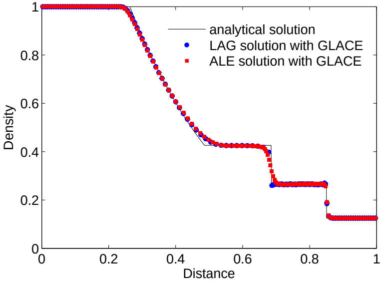
Results - Monomaterial description - Sod shock tube
Initial conditions

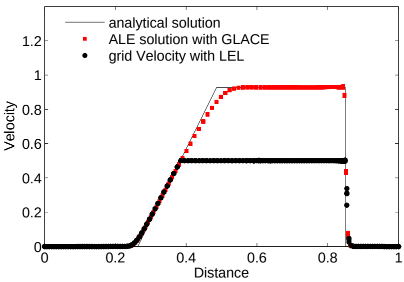


- *Mesh* : 150×15 cells
- *Lagrangian scheme* : GLACE
- *boundary conditions* : plane Wall everywhere
- *EOS* : Ideal Gas law with $\gamma = 1.4$
- *CFL* $= 0.5$
- *limiters for* $\overrightarrow{w}$: $\chi_g = 0.5$, $\chi_f = 1.0$

Sod shock tube - $t_{\text{final}} = 0.2$

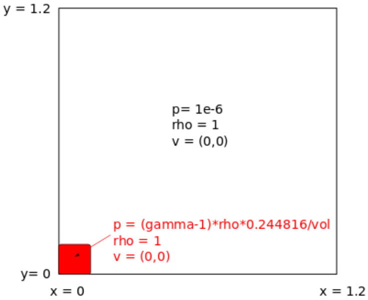






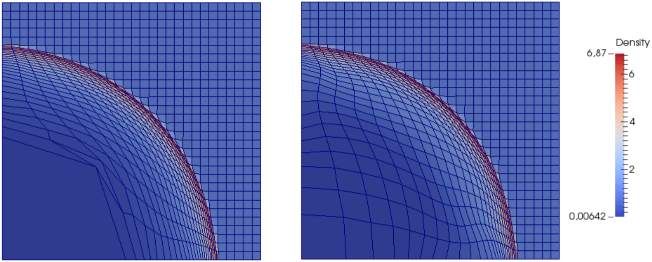
Results - Monomaterial description - Sedov Blast wave

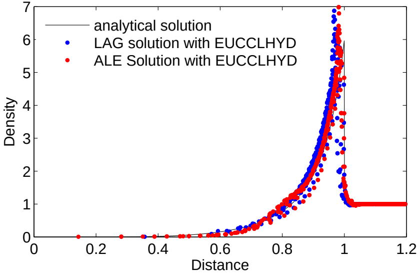
Initial Conditions



- *Mesh* : 30×30 cells
- *Lagrangian scheme* : EUCCLHYD
- *boundary conditions* : plane Wall everywhere
- *EOS* : Ideal Gas law with $\gamma = 1.4$
- *limiters for* $\overrightarrow{w}$: $\chi_g = 0.7$, $\chi_f = 1.0$

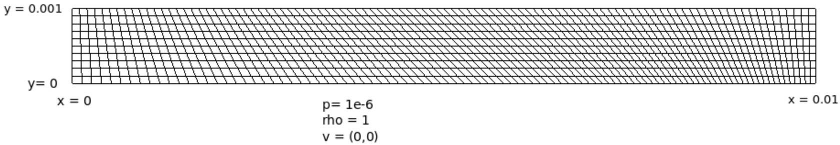
Sedov blast wave - $t_{\text{final}} = 1$





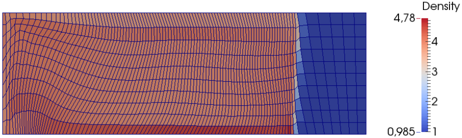
Results - Monomaterial description - Saltzman test case

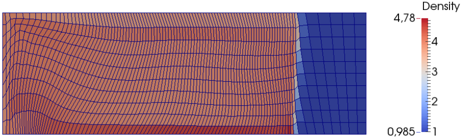
Initial Conditions

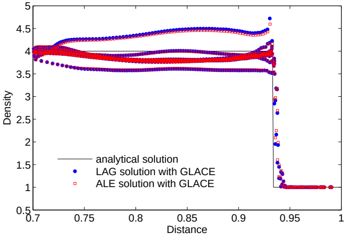


- *Mesh :* 100×10 cells
- *Lagrangian scheme :* EUCCLHYD
- *boundary conditions :* plane wall on the top, bottom and right boundaries. Normal velocity imposed on the left boundary : $\overrightarrow{v} \cdot (1,0)^T = 0.01$.
- *EOS :* Ideal Gas law with $\gamma = 5/3$
- *limiters for* $\overrightarrow{w}$: $\chi_g = 1$, $\chi_f = 0.5$

Saltzman test case - $t_{\text{final}} = 0.7$

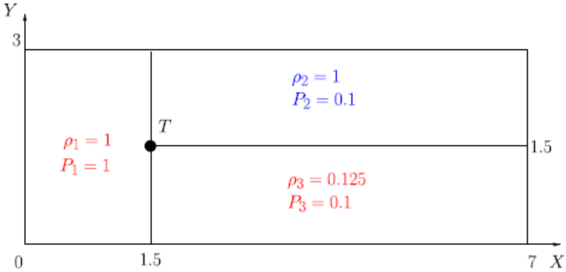






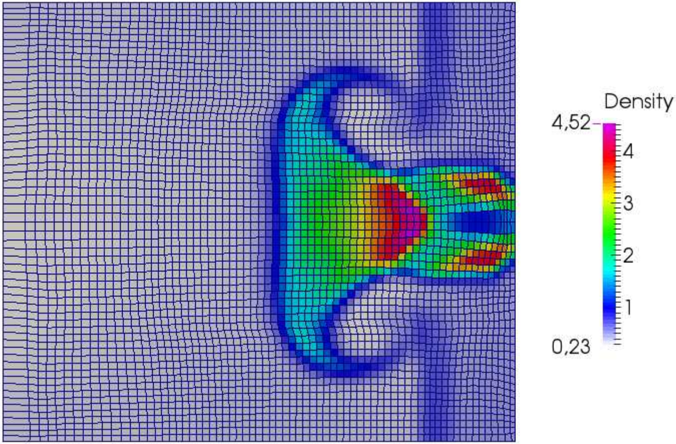
Results - Monomaterial description - Triple point

Initial Conditions



- *Mesh* : 70×30 cells
- *Lagrangian scheme* : GLACE
- *boundary conditions* : plane wall everywhere
- *EOS* : Ideal Gas law with $\gamma = 1.4$
- *limiters for* $\overrightarrow{w}$: $\chi_g = 0.1$, $\chi_f = 0.015$

Monomaterial Triple point - $t_{\text{final}} = 5s$



Physical Model - Two-material description

Description with a single velocity and a single total energy :

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \mathrm{dV}\right) + \int_{A(t)} \overrightarrow{w}(t)\cdot\overrightarrow{\nu}\ \mathrm{dS} = 0$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho^+\ \mathrm{dV}\right) + \int_{A(t)} \rho^+(t)\overrightarrow{\mu}(t)\cdot\overrightarrow{\nu}\ \mathrm{dS} = 0\,,$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho^-\ \mathrm{dV}\right) + \int_{A(t)} \rho^-(t)\overrightarrow{\mu}(t)\cdot\overrightarrow{\nu}\ \mathrm{dS} = 0\,,$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho\overrightarrow{v}\ \mathrm{dV}\right) + \int_{A(t)} \rho(t)\overrightarrow{v}(t)\ \overrightarrow{\mu}\cdot\overrightarrow{\nu}\ \mathrm{dS} + \int_{A(t)} p\overrightarrow{\nu}\ \mathrm{dS} = 0\,,$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\int_{V(t)} \rho E\ \mathrm{dV}\right) + \int_{A(t)} \rho(t)H(t)\ \overrightarrow{\mu}\cdot\overrightarrow{\nu}\ \mathrm{dS} = 0\,.$$

We also consider the following relations :

- species conservation : $\alpha^+ + \alpha^- = 1$
- mass conservation : $\alpha^+\rho^+ + \alpha^-\rho^- = \rho$
- Total energy conservation : $\alpha^+\rho^+E^+ + \alpha^-\rho^-E^- = \rho E$

Interface model

The interface $\Gamma = \Omega^+ \cap \Omega^-$ is described using a level-set representation :

$$\Gamma = \{\overrightarrow{x} \in \Omega,\ \ \phi(\overrightarrow{x}) = 0\}$$

with ϕ the signed Euclidian distance defined as :

$$\begin{aligned} \phi : \mathbb{R} &\rightarrow \mathbb{R} \\ \overrightarrow{x} \rightarrow \phi(\overrightarrow{x}) &= \begin{cases} d(\overrightarrow{x}, \Gamma) & \text{if} \quad \overrightarrow{x} \in \Omega^+ \\ -d(\overrightarrow{x}, \Gamma) & \text{if} \quad \overrightarrow{x} \in \Omega^- \\ 0 \quad \text{if} \quad \overrightarrow{x} \in \Gamma \end{cases} \end{aligned}$$

The motion of the interface Γ is given by the finite volume level-set equation :

$$\frac{\mathrm{d}}{\mathrm{dt}} \left(\int_{V(t)} \rho\phi \ \mathrm{dV} \right) + \int_{A(t)} \rho\phi \ \overrightarrow{\mu} \cdot \nu \ \mathrm{dS} = 0$$

The local normal to the interface is connected to the gradient of ϕ :
$\overrightarrow{n} = -\dfrac{\overrightarrow{\nabla}(\phi)}{||\overrightarrow{\nabla}(\phi)||}$. It is used to compute the surfaces of exchange A_k^+ and A_k^- (PLIC procedure) involved in the conservation equations.

Reinitialization step

The "distance" property of the function ϕ does not hold as time evolves. After solving the conservation equations, a reinitializing step is performed by solving the IVP on the updated mesh $\mathcal{M}^{n+1}$:

$$\int_{V(t)} \left(\frac{\partial}{\partial t} (\phi) - \phi \overrightarrow{\nabla} \cdot (\overrightarrow{w}) \right) \, \mathrm{dV} + \int_{A(t)} \phi \, \overrightarrow{d} \cdot \overrightarrow{\nu} \, \mathrm{dS} = \int_{V(t)} \mathrm{sign}(\phi_0) \, \mathrm{dV} \tag{17}$$

$$\phi_0 = \phi(\overrightarrow{x}, t^{n+1})$$

with $\overrightarrow{d} = \mathrm{sign}(\phi_0) \dfrac{\overrightarrow{\nabla}(\phi)}{||\overrightarrow{\nabla}(\phi)||}$.

Numerical Scheme - Two-material description

$$\frac{\mathrm{d}}{\mathrm{dt}}(V_j) + \sum_{k\in\mathcal{E}(j)} A_k \overrightarrow{w_k} \cdot \overrightarrow{\nu_{j,k}} = 0\,, \mathrm{GCL}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\alpha_j^+ \rho_j^+ V_j\right) + \sum_{k\in\mathcal{E}(j)} A_k^+ \rho_k^{+,\mathrm{up}}\ \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} = 0\,, \mathrm{mass}\ +$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\alpha_j^- \rho_j^- V_j\right) + \sum_{k\in\mathcal{E}(j)} A_k^- \rho_k^{-,\mathrm{up}}\ \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} = 0\,, \mathrm{mass}\ \text{-}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\rho_j V_j \overrightarrow{v_j}\right) + \sum_{k\in\mathcal{E}(j)} A_k \left(\rho_k^{\mathrm{up}} \overrightarrow{v_k^{\mathrm{up}}}\ \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} + p_{j,k}\overrightarrow{\nu_{j,k}}\right) = 0\,, \mathrm{momemtum}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\rho_j V_j E_j\right) + \sum_{k\in\mathcal{E}(j)} A_k \left(\rho_k^{\mathrm{up}} E_k^{\mathrm{up}}\ \overrightarrow{\mu_k} + p_{j,k}\ \overrightarrow{v_k}\right) \cdot \overrightarrow{\nu_{j,k}} = 0\,, \text{total energy}$$

Conservation of species, mass and total energy is expressed using the consistent relations :

$$\begin{aligned} 1 &= \alpha_j^+ + \alpha_j^-\,, \\ A_k \rho_k^{\mathrm{up}} &= A_k^+ \rho_k^{+,\mathrm{up}} + A_k^- \rho_k^{-,\mathrm{up}} \\ A_k \rho_k^{\mathrm{up}} E_k^{\mathrm{up}} &= A_k^+ \rho_k^{+,\mathrm{up}}\ E_k^{+,\mathrm{up}} + A_k^- \rho_k^{-,\mathrm{up}}\ E_k^{-,\mathrm{up}} \\ A_k &= A_k^+ + A_k^- \end{aligned}$$

Computation of the volume fraction α_j^{n+1} from ϕ_j^{n+1}

1. Solve the Semi-discrete formulation of the level set equation :

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\rho_j V_j \phi_j\right) + \sum_{k \in \mathcal{E}(j)} A_k \rho_k^{up} \phi_k^{up} \ \overrightarrow{\mu_k} \cdot \overrightarrow{\nu_{j,k}} = 0$$

   and deduce ϕ_j^{n+1} from the equation of mass.

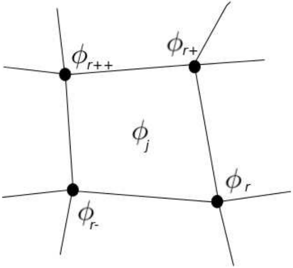
2. Then, reinitialize ϕ_j^{n+1} by seeking the stationary solution of the equation

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\phi_j V_j\right) + \sum_{k \in \mathcal{E}(j)} A_k \left(\rho_k^{up} \phi_k^{up} - \phi_j\right) \ \overrightarrow{d_k} \cdot \overrightarrow{\nu_{j,k}} = \mathrm{sign}(\phi^0) V_j$$

3. interpolate the nodal values of ϕ from the centered values using the arithmetic mean :

$$\forall r \in \mathcal{N}, \ \ \phi_r^{n+1} = \frac{1}{\mathrm{card}\left(\mathcal{C}(r)\right)} \sum_{j \in \mathcal{C}(r)} \phi_j^{n+1}$$

Computation of the volume fraction α_j^{n+1} from ϕ_j^{n+1}



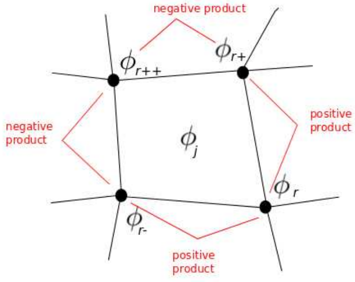
4. in a mixed cell, search the points which cancels ϕ on the surrounding edges for which $\phi_r \ \phi_{r+1} < 0$.

$$\overrightarrow{x} = (1-\beta)\,\overrightarrow{x_r} + \beta\,\overrightarrow{x_{r+1}}$$

$$\text{with } \beta = -\frac{\phi_r}{\phi_{r+1} - \phi_r}$$

5. compute $V_j^{\pm}$, $\alpha_j^{\pm}$ and finally $A_k^{\pm}$ from the set of nodes.

Computation of the volume fraction α_j^{n+1} from ϕ_j^{n+1}



4. in a mixed cell, search the points which cancels ϕ on the surrounding edges for which $\phi_r\ \phi_{r+1} < 0$.

$$\overrightarrow{x} = (1-\beta)\ \overrightarrow{x_r} + \beta\ \overrightarrow{x_{r+1}}$$

$$\text{with}\ \ \beta = -\frac{\phi_r}{\phi_{r+1} - \phi_r}$$

5. compute $V_j^{\pm}$, $\alpha_j^{\pm}$ and finally $A_k^{\pm}$ from the set of nodes.

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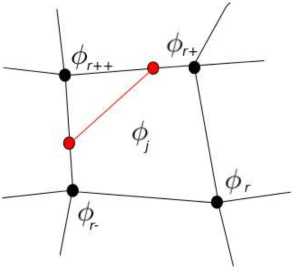

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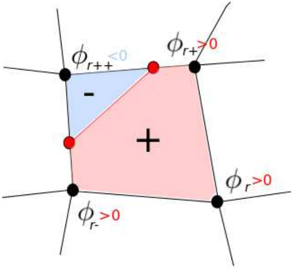

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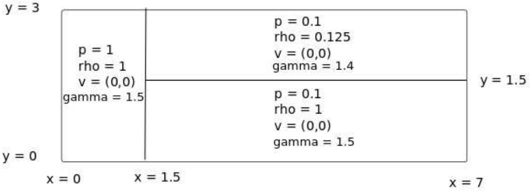
Results - Two-material Triple point

Initial Conditions



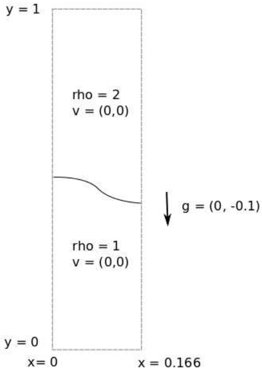
- *Mesh* : 280 × 120 cells
- *Lagrangian scheme* : GLACE
- *boundary conditions* : plane wall everywhere
- *EOS* : Ideal Gas law
- *limiters for* $\overrightarrow{w}$: $\chi_g = 0$, $\chi_f = 0$. (Eulerian Framework)

Two-material Triple point. $t_{\text{final}} = 5$

click here to start

FIGURE : Density map for the multimaterial triple point test case in the Eulerian framework. The white line denotes the interface.

Results - Multimaterial description - Rayleigh-Taylor instability
Initial Conditions



- *Mesh* : 32×160 cells
- *Lagrangian scheme* : GLACE
- *boundary conditions* : plane wall everywhere
- *EOS* : Ideal Gas law with $\gamma = 1.4$.
- *limiters for* $\overrightarrow{w}$: $\chi_g = 0$, $\chi_f = 0$. (Eulerian Framework)

Two-material Rayleigh-Taylor instability. $t_{\text{final}} = 10$

click here to start

FIGURE : Density map for the multimaterial Rayleigh-Taylor test case in the Eulerian framework. The white line denotes the interface.

Conclusions and perspectives

- Our numerical scheme seems to be robust and accurate.
- A lot of work to do for the multimaterial description.
- The global computational method for the grid velocity is inefficient for complex flows that are composed of both vortices and shocks. A local method should be used instead.
- The extension to the case $nbmat \geq 3$ is in progress. The main idea is to use $nbmat - 1$ level set equations for each interface, the main problem being the reinitializing step that may be much more complicated (and time consuming).

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