A cell-centered Finite Volume method for solving multi-dimensional hyper-elasticity equations written under total Lagrangian form

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ABSTRACT

Finite Volume discretization of non-linear elasticity equations seems to be a promising alternative to the traditional Finite Element discretizations currently employed [1]. In this work, we intend to derive a Finite Volume discretization to compute the large deformations of elastic isotropic isothermal materials including shock waves. The materials under consideration are characterized by an hyper-elastic constitutive law for which the second Piola Kirchhoff stress tensor is the derivative of a strain energy function with respect to the right Cauchy Green tensor. In this manner, the material model satisfies the principle of material frame indifference and is thermodynamically consistent. The set of governing equations consists of the linear momentum equation, which is written under total Lagrangian form, and the geometric conservation law (GCL), which is nothing but the time rate of change of the deformation gradient. Extending the methodology described in [2] for Lagrangian gas dynamics, we present a nominally second-order cell-centered discretization for solving the equations of non-linear elasticity on tetrahedral grids. This work can also be viewed as an extension of the two-dimensional first-order scheme introduced in [3]. We provide a spatial discretization of the deformation gradient for which the Piola compatibility condition is satisfied. Further, the numerical fluxes at cell interfaces are approximated to ensure a thermodynamic-like dissipation inequality. The robustness and the accuracy of this Finite Volume method for non-linear elasticity are assessed by means of numerous representative test cases.

References

[1] C.H. Lee and A.J. Gil and J. Bonet, Development of a cell centered upwind finite volume algorithm for a new conservation law formulation in structural dynamics, *Computers and Structures*, 118 (2013) 13-38.

[2] P.-H. Maire and R. Abgrall and J. Breil and R. Loubère and B. Rebourcet, A nominally second-order cell-centered Lagrangian scheme for simulating elastic-plastic flows on two-dimensional unstructured grids, *Journal of Computational Physics*, 235 (2013) 626-665.

[3] G. Kluth and B Després. Discretization of hyper-elasticity on unstructured mesh with a cell-centered Lagrangian scheme *Journal of Computational Physics*, 229 (2010) 9092-9118.

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