Direct Arbitrary-Lagrangian-Eulerian ADER-MOOD Finite Volume Schemes for Multidimensional Hyperbolic Conservation Laws

W. Boscheri[†], **Raphaël Loubère**[‡] **and M. Dumbser**[†]

[†] Laboratory of Applied Mathematics, Department of Civil, Environmental and Mechanical Engineering, University of Trento, Via Mesiano, 77 - 38123 Trento, Italy
(walter.boscheri@unitn.it, michael.dumbser@unitn.it)

[‡] CNRS and Institut de Mathématiques de Toulouse (IMT) Université Paul-Sabatier, Toulouse, France
(raphael.loubere@math.univ-toulouse.fr)

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ABSTRACT

We present a new family of efficient high order accurate direct Arbitrary-Lagrangian-Eulerian (ALE) one-step ADER-MOOD finite volume schemes for the solution of nonlinear hyperbolic systems of conservation laws for moving unstructured triangular and tetrahedral meshes. This family is the next generation of the ALE ADER-WENO schemes presented in [1, 2]. Here, we use again an element-local space-time Galerkin finite element predictor method to achieve a high order accurate one-step time discretization, while the somewhat expensive WENO approach on moving meshes, used to obtain high order of accuracy in space, is replaced by an *a posteriori* MOOD loop which is shown to be less expensive but still as accurate. This *a posteriori* MOOD loop ensures the numerical solution in each cell at any discrete time level to fulfill a set of user-defined detection criteria. If a cell average does not satisfy the detection criteria, then the solution is locally re-computed by progressively decrementing the order of the polynomial reconstruction, following a so-called *cascade* of predefined schemes with decreasing approximation order. A so-called parachute scheme, typically a very robust first order Godunov-type finite volume method, is employed as a last resort for highly problematic cells. The cascade of schemes defines how the decrementing process is carried out, i.e. how many schemes are tried and which orders are adopted for the polynomial reconstructions. The cascade and the parachute scheme are choices of the user or the code developer. Consequently the iterative MOOD loop allows the numerical solution to maintain some interesting properties such as positivity, mesh validity, *etc*, which are otherwise difficult to ensure. We have applied our new high order unstructured direct ALE ADER-MOOD schemes to the multi-dimensional Euler equations of compressible gas dynamics. A large set of test problems has been simulated and analyzed to assess the validity of our approach in terms of both accuracy and efficiency (CPU time and memory consumption).

References

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