

# Scalable High-order ALE Simulations

R. Anderson, T. Brunner, V. Dobrev, I. Karlin, T. Kolev, R. Rieben<sup>†</sup>, V. Tomov

<sup>†</sup> Lawrence Livermore National Lab ([rieiben1@llnl.gov](mailto:rieiben1@llnl.gov))

## ABSTRACT

The Arbitrary Lagrangian-Eulerian (ALE) framework forms the basis of many large-scale multi-physics codes, and in particular those centered around radiation diffusion and shock hydrodynamics. Current ALE discretization approaches consist of a Lagrange phase, where the hydrodynamics equations are solved on a moving mesh, followed by a three-part “advection phase” involving mesh optimization, field remap and multi-material zone treatment. While traditional low-order ALE methods have been successful at extending the capability of pure Lagrangian methods, they also introduce numerical problems of their own including breaking of symmetry, and lack of energy conservation.

In this talk, we present a general high-order finite element discretization framework that aims to improve the quality of current ALE simulations of radiation-hydrodynamics, while also improving their performance on modern data-centric computing architectures. We use the de Rham complex to guide the discretization of different physics components. In particular, kinematic quantities (e.g. velocity, position) are discretized with continuous (H1) finite elements, thermodynamic quantities (e.g. internal energy) use continuous (L2) elements, while H(div)-conforming finite elements are used for the fluxes in radiation diffusion.

Our Lagrangian hydrodynamics algorithm is based on Galerkin variational formulation of momentum and energy conservation using the high-order de Rham finite elements. The use of high-order position description enables curvilinear zone geometries allowing for better approximation of the mesh curvature, which develops naturally with the flow. The remap phase of ALE is posed as an advection problem in artificial pseudo-time, describing the evolution of the post-Lagrangian mesh into the improved new mesh. This is discretized using a finite element Discontinuous Galerkin (DG) approach on high-order curvilinear meshes. The semi-discrete DG method results in high-order accuracy for sufficiently smooth fields, but can produce non-monotonic results for discontinuous fields. We consider several non-linear approaches to enforce monotonicity, including high-order algebraic Locally Scaled Diffusion (LSD), Flux Corrected Transport (FCT) and Optimization Based Remap (OBR). The ALE evolution of different materials in our framework uses high-order “material indicator” functions, and we have developed high-order closure models to model the sub-zonal material behavior during the Lagrangian phase.

We have started exploring approaches for discretizing high-order multi-group radiation diffusion on general curvilinear grids and will report some initial results on the coupling with the high-order hydrodynamics. We will also present numerical tests illustrating the robustness and scalability of our discretization algorithms and discuss recent work to further improving their performance on modern architectures.

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